CALCULATION OF GEAR RATING
FOR MARINE TRANSMISSIONS

JULY 1993
FOREWORD

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Classification, certification and quality assurance of ships, offshore installations and industrial plants, as well as testing and certification of materials and components, are main activities.

Det Norske Veritas possesses technological capability in a wide range of fields, backed by extensive research and development efforts. The organization is represented worldwide in more than 100 countries.

Classification Notes are publications which give practical information on classification of ships, offshore installations and other objects. Examples of design solutions, calculation methods, specifications of test procedures, quality assurance and quality control systems as well as acceptable repair methods for some components are given as interpretations of the more general rule requirements.

An updated list of Classification Notes is available on request. The list is also given in the latest edition of the Introduction-booklets to the «Rules for Classification of Ships», the «Rules for Classification of Mobile Offshore Units» and the «Rules for Classification of High Speed and Light Craft».

In «Rules for Classification of Fixed Offshore Installations», only those Classification Notes which are relevant for this type of structure have been listed.

It has been assumed in the drafting of this Classification Note that the execution of its provisions is entrusted to appropriately qualified and experienced people, for whose use it has been prepared.

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1. Basic Principles and General Influence Factors

1.1 Scope and Basic Principles

The gear rating procedures given in this Classification Note are mainly based on the ISO-DIS 6336 Part 1—5 (cylindrical gears), and ISO-DIS 10300 Part 1—3 (bevel gears) and drafts for ISO Technical Reports on Scuffing and Fatigue Damage Accumulation, but specially applied for marine purposes, such as marine propulsion and important auxiliarys on board ships and mobile offshore units.

The calculation procedures cover gear rating as limited by contact stresses (pitting, spalling or case crushing), tooth root stresses (fatigue breakage or overload breakage), and scuffing resistance. Even though no calculation procedures for other damages such as wear, gray staining (micropitting), fractures starting from flanks, etc. are given, such damages may limit the gear rating. Enclosed parallel shaft gears, epicyclic gears and bevel gears (with intersecting axes) for infinite life or limited life, are applicable. Steel is the only material considered. The methods applied throughout this document are only valid for a transverse contact ratio 1 < e < 2. If this does not apply, special considerations are to be made.

All influence factors are defined regarding their physical interpretation. Some of the influence factors are determined by the gear geometry or have been established by conventions. These factors are to be calculated in accordance with the equations provided. Other factors are approximations, which is clearly stated in the text by terms as «may be calculated as». These approximations are substitutes for exact evaluations where such are lacking or too extensive for practical purposes, or factors based on experience. In principle, any suitable method may replace these approximations.

Bevel gears are calculated on basis of virtual (equivalent) cylindrical gears using the geometry of the midsurface. The virtual (helical) cylindrical gear is to be calculated by using all the factors as a real cylindrical gear with some exceptions. These exceptions are mentioned in connection with the applicable factors. Wherever a factor or calculation procedure has no reference to either cylindrical gears or bevel gears, it is generally valid, i.e. combined for both cylindrical and bevel. In order to minimize the volume of this Classification Note such combinations are widely used, and everywhere it is necessary to distinguish, it is clearly pointed out by local headings:

**Cylindrical gears**

**Bevel gears**

The permissible contact stresses, tooth root stresses and scuffing load capacity depend on the safety factors as required in the respective Rule sections. Terms as endurance limit and static strength are used throughout this Classification Note. Endurance limit is to be understood as the fatigue strength in the range of cycles beyond the lower knee of the σ-N curves, regardless if it is constant or drops with higher number of cycles.

Static strength is to be understood as the fatigue strength in the range of cycles less than at the upper knee of the σ-N curves.

For gears which are subjected to a limited number of cycles at different load levels, a cumulative fatigue calculation applies. Information on this is given in S (Appendix A).

When the term infinite life is used, it means number of cycles in the range 10⁸—10¹⁸.

1.2 Symbols, Nomenclature and Units

The symbols are generally from ISO 701-1976, ISO/R31 and ISO 1328-1975, with a few additional symbols. Only SI units are used.

The main symbols as influence factors (K, Z, Y and X with indexes) etc. are presented in their respective headings. Symbols which are not explained in their respective Sects. are as follows:

\[ a = \text{centre distance (mm).} \]
\[ b = \text{facewidth (mm).} \]
\[ d = \text{reference diameter (mm).} \]
\[ da = \text{tip diameter (mm).} \]
\[ db = \text{base diameter (mm).} \]
\[ dw = \text{working pitch diameter (mm).} \]
\[ h_a = \text{addendum (mm).} \]
\[ h_0 = \text{addendum of tool ref. to } m_a. \]
\[ h_f = \text{bounding moment arm (mm) for tooth root stresses for application of load at the outer point of single tooth pair contact.} \]
\[ h_{fa} = \text{bounding moment arm (mm) for tooth root stresses for application of load at tooth tip.} \]
\[ H_B = \text{Bend hardness.} \]
\[ H_V = \text{Vickers hardness.} \]
\[ H_R = \text{Rohr hardness.} \]
\[ H_{RC} = \text{Rockwell C hardness.} \]
\[ m_n = \text{normal module.} \]
\[ m = \text{rev. per minute.} \]
\[ N_L = \text{number of load cycles.} \]
\[ p_{ro} = \text{proportionality value ref. to } m_a. \]
\[ h_a = \text{notch parameter.} \]
\[ h_r = \text{average roughness value (µm).} \]
\[ R_s = \text{peak to valley roughness (µm).} \]
\[ R_y = \text{average peak to valley roughness (µm).} \]
\[ s_{fn} = \text{tooth root chord (mm) in the critical section.} \]
\[ T = \text{torque (Nm).} \]
\[ u = \text{gear ratio (per stage).} \]
\[ v = \text{linear speed (m/s) at pitch diameter.} \]
\[ x = \text{addendum modification coefficient.} \]
\[ z = \text{number of teeth.} \]
\[ \alpha_k = \text{virtual number of spur teeth.} \]
\[ \alpha = \text{normal pressure angle at ref. cylinder.} \]
\[ \alpha_t = \text{transverse pressure angle at ref. cylinder.} \]
\[ \alpha_w = \text{transverse pressure angle at pitch cylinder.} \]
\[ \beta = \text{helix angle at base cylinder.} \]
\[ \beta_h = \text{helix angle at base cylinder.} \]
\[ \tau_{e_s} = \text{transverse contact ratio.} \]
\[ \tau_{g} = \text{overlap ratio.} \]
\[ \tau_{9} = \text{total contact ratio.} \]
\[ \rho_{t} = \text{tip radius of tool ref. to } m_a. \]
\[ \rho_{r} = \text{root radius of basic rack ref. to } m_n (= \rho_{m}). \]
\[ \rho_C = \text{effective radius (mm) of curvature at pitch point.} \]
\[ \rho_{r} = \text{root fillet radius (mm) in the critical section.} \]
\[ \sigma_b = \text{ultimate tensile strength (N/mm²).} \]
\[ \sigma_f = \text{yield strength resp 0.2% proof stress (N/mm²).} \]

Index 1 refers to the pinion, 2 to the wheel.

Special additional symbols for bevel gears are as follows:

\[ \Sigma = \text{angle between intersecting axes.} \]
\( \delta \) = pitch cone angle.

\( \chi_m \) = tooth thickness modification coefficient (midface).

\( R \) = pitch cone distance (mm).

Index \( v \) refers to the virtual (equivalent) helical cylindrical gear.

Index \( m \) refers to the midsection of the bevel gear.

### 1.3 Geometrical Definitions

For internal gearing \( z_2, a; d_{s2}, d_{w2}, d_2 \) and \( d_{b2} \) are negative, \( x_2 \) is positive if \( d_{s2} \) is increased, i.e. the numeric value is decreased.

The pinion has the smaller number of teeth, i.e.

\[ |u| = \frac{|z_2|}{z_1} \geq 1 \]

For calculation of surface durability \( b \) is the common facewidth on pitch diameter.

For tooth strength calculations \( b_1 \) or \( b_2 \) are facewidths at the resp. tooth roots. If \( b_1 \) or \( b_2 \) differ much from \( b \) above, they are not to be taken more than 1 module on either side of \( b \). However, if crowning or end relief is chosen so that the hardest contact is not at the end of the facewidth, then \( b \) is to be used for both pinion and wheel tooth root stresses.

**Cylindrical gears**

\( \tan \alpha_t = \frac{\tan \alpha_1 \cos \beta}{\cos \alpha_1} \)

\( \tan \beta_b = \frac{\tan \beta \cos \alpha_1}{\cos \beta} \)

\( d = z_1 m_1 \cos \beta \)

\( d_b = (d \cos \alpha_1 = d_w \cos \alpha_{tw}) \)

\( a = 0,5 (d_1 + d_2) \)

\( d_{w1}/d_{w2} = z_1/z_2 \)

\( \text{inv} \alpha = \tan \alpha - \alpha \) (radians)

\( \text{inv} \alpha_{\text{tw}} = \text{inv} \alpha + 2 \tan \alpha_2 (x_1 + x_2)/(x_1 + x_2) \)

\( n_0 = n (\cos^2 \beta \cos \beta) \)

\[ \epsilon_a = \frac{0,5 \sqrt{d_{b1}^2 - d_{b1}^2} + 0,5 \sqrt{d_{b2}^2 - d_{b2}^2} - \sin \alpha_{\text{tw}}}{\pi m_1 \cos \alpha_1 \cos \beta} \]

\[ + \text{ if external gear pair} \]

\[ - \text{ if internal gear pair} \]

\( \epsilon_\beta = \frac{b \sin \beta}{\pi m_1} \)

(for double helix, \( b \) is to be taken as the width of one helix).

\( \epsilon_\gamma = \epsilon_x + \epsilon_\rho \)

\( \rho C = \frac{\alpha \sin \alpha_2 \mu}{\cos \beta_1 (1 + \mu)^2} \)

\( \nu = \frac{\pi}{60} \alpha_1 d_1 \times 10^{-3} \)

\( \rho_{\text{min}} = \frac{\pi m_1 \cos \alpha_1}{\cos \beta} \)

### 1.4 Bevel Gear Conversion Formulae and Specific Formulae

Conversion of bevel gears to virtual equivalent helical cylindrical gears is based on the bevel gear midsection. The conversion formulae are:

- **Number of teeth:**
  \[ z_{v1,2} = z_{1,2} \frac{\cos \delta_1,2}{(\delta_1 + \delta_2 - \Sigma)} \]

- **Gear ratio:**
  \[ u_v = \frac{z_{v2}}{z_{v1}} \]

- **Base pitch:**
  \[ p_{\text{min}} = \frac{\pi m_1 \cos \alpha_\text{tw}}{\cos \beta_{\text{tw}}} \]

- **Reference, pitch, diameters:**
  \[ d_{v1,2} = \frac{d_{m1,2}}{\cos \delta_{1,2}} \]

- **Centre distance:**
  \[ a_v = 0,5 (d_{v1} + d_{v2}) \]

- **Tip diameters:**
  \[ d_{a1,2} = d_{v1,2} + 2 h_{\text{m1,2}} \]

- **Addenda:**
  - for gears with constant addenda (Klingelnberg):
    \[ h_{\text{m1,2}} = m_{\text{m1}} (1 + \chi_{1,2}) \]
  - for gears with variable addenda (Gleason):
    \[ h_{\text{m1,2}} = h_{a1,2} - b/2 \tan (\delta_{1,2} - \delta_a) \]
    (when \( h_a \) is addendum at outer end and \( \delta_a \) is the outer cone angle).

- **Addendum modification coefficients:**
  \[ \chi_{1,2} = \frac{h_{\text{m1,2}} - h_{a1,2}}{2 m_{\text{m1}}} \]

- **Base circle:**
  \[ d_{v1,2} = d_{v1,2} \cos \alpha_\text{tw} \]

- **Transverse contact ratio:**
  \[ \xi_{\text{tw}} = \frac{0,5 \sqrt{d_{w1}^2 - d_{w1}^2} + 0,5 \sqrt{d_{w2}^2 - d_{w2}^2} - a_v \sin \alpha_\text{tw}}{p_{\text{min}}} \]

- **Overlap ratio:**
  \[ \xi_\rho = \frac{b \sin \beta_{\text{tw}}}{\pi m_{\text{m1}}} \]

- **Total contact ratio:**
  \[ t_\gamma = \sqrt{t_\delta^2 + \epsilon_\rho^2} \]
(Note that index $w$ is left out in order to combine formulae for cylindrical and bevel gears)

Tangential speed at midsection:

$$v_{ml} = \frac{\pi n_1 d_{ml}}{60} 10^{-3}$$

Effective radius of curvature (normal section)

$$\rho_{vn} = \frac{a_v \sin \alpha_v \nu_v}{\cos \beta_{vn} (1 + u_v)^2}$$

Length of line of contact:

$$l_b = \frac{b_\epsilon_\beta \sqrt{e^2 - (2 - e_\beta) (1 - e_\beta)^2}}{e_\beta}$$

$e = \left(1 + \frac{\rho_{vn}}{\rho_{sn}}\right)$ if $e_\beta < 1$

$$l_b = \frac{b_\epsilon_\beta}{e_\epsilon \cos \beta_{sn}}$$

if $e_\beta \geq 1$

### 1.5 Nominal Tangential Load, $F_t$, $F_{ht}$, $F_m$, and $F_{net}$

The nominal tangential load (tangential to the reference cylinder with diameter $d$ and perpendicular to an axial plane) is calculated from the nominal torque $T$ transmitted by the gear set.

**Cylindrical gears**

$$F_t = \frac{2000 T}{d}$$

**Bevel gears**

$$F_{ht} = \frac{F_t}{\cos \alpha_t}$$

$$F_m = \frac{2000 T}{d_m}$$

$$F_{net} = \frac{F_m}{\cos \alpha_v}$$

### 1.6 Application Factors, $K_A$ and $K_{AP}$

The application factor $K_A$ accounts for dynamic overloads from sources external to the gearing.

It is distinguished between the influence of repetitive cyclic torques $K_A$ (1.6.1) and the influence of temporary occasional peak torques $K_{AP}$ (1.6.2).

Calculations are always to be made with $K_A$. In certain cases additional calculations with $K_{AP}$ may be necessary.

#### 1.6.1 $K_A$

For gears designed for long or infinite life at nominal rated torque, $K_A$ is defined as the ratio between the maximum repetitive cyclic torque applied to the gear set and nominal rated torque.

This definition is suitable for main propulsion gears and most of the auxiliary gears.

$K_A$ can be determined by measurements or system analysis, or may be ruled by conventions (ice classes). (For the purpose of a preliminary (but not binding) calculation before $K_A$ is determined, it is advised to apply either the max. values mentioned below or values known from similar plants.)

- **a)** For main propulsion gears $K_A$ can be taken from the (mandatory) torsional vibration analysis, thereby considering all permissible driving conditions.
  
  Guidance: Unless specially agreed, the rules do not allow $K_A$ in excess of 1.5 for diesel propulsion. With turbine or electric propulsion $K_A$ would normally not exceed 1.2.

- **b)** For main propulsion gears with ice class notation (see Rules Pt.3 Ch.1 Sec J500) $K_A$ has to be taken as the higher value of the applicable (rule defined) ice shock load to nominal rated torque and the value under a).

The **baltic** ice class notation refers to approx. $10^6$ ice shock loads.

Additionally, the calculations with the normal $K_A$ (no ice class) are to fulfill the normal requirements.

For **polar** ice class notations, $K_A$ applies to all criteria and for long or infinite life.

- **c)** For diesel driven auxiliaries $K_A$ can be taken from the torsional vibration analysis, if available. For units where no vibration analysis is required ($<200$ kW) or available, it is advised to apply $K_A$ as the upper allowable value 1.5.

- **d)** For turbine or electro driven auxiliaries the same as for c) applies, however the practical upper value is 1.2.

#### 1.6.2 $K_{AP}$

The peak overload factor $K_{AP}$ is defined as the ratio between the temporary occasional peak overload torque and the nominal rated torque.

For plants where high temporary occasional peak torques can occur (i.e. in excess of the above mentioned $K_A$), the gear has to be checked with regard to static strength. Unless otherwise specified the same safety factors as for infinite life apply. The scuffing safety is to be specially considered, whereby the $K_A$ applies in connection with the bulk temperature, and the $K_{AP}$ applies for the flash temperature calculation and should replace $K_A$ in the formulae in 4.3.1, 4.3.2 and 4.4.1.

$K_{AP}$ can be taken from the additional ice class notation, unless specially approved.

For plants **without** additional ice class notation, $K_{AP}$ should normally not exceed:

- electric prime mover, 1.5
- driven generator, 2.0
- rapidly engaging clutch, 1.5
- any kind of prime mover or driven machine, unless specially approved, 2.0

#### 1.6.3 Frequent overloads

For plants where high overloads or shockloads occur regularly, the influence of this is to be considered by means of cumulative fatigue. (See Appendix A). If the overloads have a duration corresponding to several revolutions of the shafts, the scuffing safety has to be considered on basis of this overload, both with respect to bulk and flash temperature. This applies to plants with ice class notations (baltic and polar), and to plants with prime movers which have high temporary overload capacity such as e.g. electric motors (provided the driven member can have a considerable increase in demand torque as e.g. azimuth thrusters).

### 1.7 Load Sharing Factor, $K_Y$

The load sharing factor $K_Y$ accounts for the maldistribution of load in multiple-path transmissions (dual tandem, epicyclic, double helix etc.). $K_Y$ is defined as the ratio between the max. load through an actual path and the evenly shared load.

$K_Y$ mainly depends on accuracy and flexibility of the branches (e.g. quill shaft, planet support, external forces...
etc.), and should be considered on basis of measurements or of relevant analysis.

If no such relevant analysis is available the following may apply:

For epicyclic gears
\[ K_y = 1 + 0.25 \sqrt{3} \]
when \( n_{pl} \) = number of planets (\( \geq 3 \))

For dual tandem gears
\[ K_y = 1 + (0.2/\phi) \]
when \( \phi \) = quill shaft twist (degrees) under full load.

For double helical gears
\[ K_y = 1 + \frac{F_{ext}}{F_t \tan \beta} \]
where \( F_{ext} \) = external axial force applied from sources outside the actual gearing.

1.8 Dynamic Factor, \( K_y \)

The dynamic factor \( K_y \) accounts for the internally generated dynamic loads.

\( K_y \) is defined as the ratio between the maximum load which dynamically acts on the tooth flanks and the maximum externally applied load \( F_t K_A K_y \).

The necessary extent and complexity of analysis to determine \( K_y \) increases with the speed. In the following 3 different methods (1.8.1, 1.8.2 and 1.8.3) are described. In case of controversy between the methods, the next following is decisive, i.e. the methods are listed with increasing priority.

It is important to observe the limitations for the methods in 1.8.1 and 1.8.2. In particular the influence of lateral stiffness of shafts is often underestimated and resonances occur at considerably lower speeds than determined in 1.8.2.1.

1.8.1 Simplified method

For slow speed gears (\( v < 10 \text{ m/s} \)) where \( K_y \) has no great significance, the simple approximation may apply:

\[ K_y = 1 + \frac{4 Q^2}{(2 + \sqrt{6}) \times 10^4} \sqrt{1 + u^2} \]

where \( Q \) is the grade of accuracy according to ISO 1328-1975 (resp. DIN 3965 for bevel gears).

It is not advised to apply this approximation for \( v z_1 > 500 \) and in general not if more accurate values are wanted.

For bevel gears the real \( z_1 \) and \( u \) (not the equivalent) and \( v \) for midface should be inserted.

The grade of accuracy \( Q \) may be taken as 6 for ground or hard metal hobbed and 8 for only lapped bevel gears.

1.8.2 Single resonance method

For single stage gears \( K_y \) may be determined on basis of the relative proximity \( N \) between actual speed \( n_1 \) and the lowest resonance speed \( n_{E1} \):

\[ N = \frac{n_1}{n_{E1}} \]

1.8.2.1 Determination of critical speed

It is not advised to apply this method for multimesh gears for \( N > 0.85 \), as the influence of higher modes has to be considered, see 1.8.3. In case of significant lateral shaft flexibility (e.g. overhung mounted bevel gears), the influence of coupled bending and torsional vibrations should be considered, see 1.8.3 if \( N \geq 0.75 \).

\[ n_{E1} = \frac{30 \times 10^3}{\pi z_1} \sqrt{\frac{c_y}{m_{red}}} \]

where \( c_y \) is the actual mesh stiffness per unit facewidth, see 1.11.

For gears with inactive ends of the facewidth, as e.g. due to high crowning or end relief such as often applied for bevel gears, the use of \( c_y \) in connection with determination of natural frequencies may need correction. \( c_y \) is defined as stiffness per unit facewidth, but when used in connection with the total mesh stiffness, it is not as simple as \( c_y b \), as only a part of the facewidth is active. Such corrections are given in 1.11.

\( m_{red} \) is the reduced mass of the gear pair, per unit facewidth and referred to the plane of contact.

For a single gear stage where no significant inertias are closely connected to neither pinion nor wheel, \( m_{red} \) is calculated as:

\[ m_{red} = \frac{m_1 m_2}{m_1 + m_2} \]

The individual masses per unit facewidth are calculated as

\[ m_{1,2} = \frac{l_{1,2}}{h (db_1/2)^2} \]

where \( l \) is the polar moment of inertia (kgm^2) (the inertia of bevel gears may be approximated as discs with diameter equal the midface pitch diameter.)

If a significant inertia (e.g. a clutch) is very rigidly connected to the pinion or wheel, it should be added to that particular inertia (pinion or wheel). If there is a shaft piece between these inertias, the torsional shaft stiffness alters the system into a 3-mass (or more) system. This can be calculated as in 1.8.2, but also simplified as a 2-mass system calculated with only pinion and wheel masses.

1.8.2.2 Factors used for determination of \( K_y \)

Non-dimensional gear accuracy dependent parameters:

\[ B_l = \frac{c^l (f_1 - y^l)}{F_t K_A K_y b} \]

Non-dimensional tip relief parameter:

\[ B_k = \left| 1 - \frac{C_a}{C_{eff}} \right| \]

For gears of quality grade (ISO 1328-1975) 6 or coarser, \( B_k = 1 \).
For gears with \( Q < 6 \) and excessive tip relief, \( B_k \) is limited to max. 1.

For gears (all quality grades) with tip relief of more than twice the adequate value (see 1.10) the reduction of \( \varepsilon_0 \) has to be considered (see 4.3).

where:

\[
\begin{align*}
\varepsilon_{p0} &= \text{the base pitch error (ISO 1328-1975), max. of pinion or wheel} \\
f_r &= \text{the total profile error (ISO 1328-1975), max. of pinion or wheel (Note: \( f_r \) is p.t. not available for bevel gears, thus use \( f_r = f_{p0} \)} \\
\varepsilon_p \text{ and } \varepsilon_r &= \text{the respective running-in allowances and may be calculated similarly to \( \varepsilon_0 \) in 1.12, i.e. the value of \( \varepsilon_{p0} \) is replaced by \( f_r \) for \( \varepsilon_r \)} \\
\varepsilon' &= \text{the single tooth stiffness, see 1.11} \\
C_a &= \text{the amount of tip relief, see 4.3.3. In case of different tip relief on pinion and wheel, the value which results in the greater value of \( B_k \) is to be used. If \( C_a \) is zero by design, the value of running-in tip relief \( C_{a'} \) (see 1.12) may be used in the above formula.}
\end{align*}
\]

\( C_{a'} \) see 4.3.2.

1.8.2.3 \( K_v \) in the subcritical sector:

For cylindrical gears: \( N \leq 0.85 \)

For bevel gears: \( N \leq 0.75 \)

\[
K_v = 1 + N K
\]

\[K = C_{v1} B_p + C_{v2} B_f + C_{v3} B_k\]

\( C_{v1} \) accounts for the pitch error influence

\[
C_{v1} = 0.32
\]

\( C_{v2} \) accounts for profile error influence

\[
C_{v2} = 0.34 \quad \text{for } \varepsilon_r \leq 2
\]

\[
C_{v2} = \frac{0.57}{\varepsilon_r - 0.3} \quad \text{for } \varepsilon_r > 2
\]

\( C_{v3} \) accounts for the cyclic mesh stiffness variation

\[
C_{v3} = 0.23 \quad \text{for } \varepsilon_r \leq 2
\]

\[
C_{v3} = \frac{0.096}{\varepsilon_r - 1.56} \quad \text{for } \varepsilon_r > 2
\]

1.8.2.4 \( K_v \) in the critical sector:

For cylindrical gears: \( 0.85 < N \leq 1.15 \)

For bevel gears: \( 0.75 < N \leq 1.25 \)

Running in this range should preferably be avoided, and is only allowed for high precision gears.

\[
K_v = 1 + C_{v4} B_p + C_{v5} B_f + C_{v6} B_k
\]

\( C_{v4} \) accounts for the resonance condition with the cyclic mesh stiffness variation

\[
C_{v4} = 0.90 \quad \text{for } \varepsilon_r \leq 2
\]

\[
C_{v4} = \frac{0.57 - 0.05 \varepsilon_r}{\varepsilon_r - 1.44} \quad \text{for } \varepsilon_r > 2
\]

1.8.2.5 \( K_v \) in the supercritical sector:

For cylindrical gears: \( N \geq 1.5 \)

For bevel gears: \( N \geq 1.5 \)

Special care should be taken as to influence of higher vibration modes, and/or influence of coupled bending (i.e. lateral shaft vibrations) and torsional vibrations. These influences are not covered by the following approach.

\[
K_v = C_{v5} B_p + C_{v6} B_f + C_{v7}
\]

\( C_{v5} \) accounts for the pitch error influence

\[
C_{v5} = 0.47
\]

\( C_{v6} \) accounts for the profile error influence

\[
C_{v6} = 0.47 \quad \text{for } \varepsilon_r \leq 2
\]

\[
C_{v6} = \frac{0.12}{\varepsilon_r - 1.74} \quad \text{for } \varepsilon_r > 2
\]

\( C_{v7} \) relates the maximum externally applied tooth loading to the maximum tooth loading of ideal, accurate gears operating in the supercritical speed sector, when the circumferential vibration becomes very soft.

\[
C_{v7} = 0.75 \quad \text{for } \varepsilon_r \leq 1.5
\]

\[
C_{v7} = 0.125 \sin[\pi(\varepsilon_r - 2)] + 0.875 \quad \text{for } 1.5 < \varepsilon_r \leq 2.5
\]

\[
C_{v7} = 1.0 \quad \text{for } \varepsilon_r > 2.5
\]

1.8.2.6 \( K_v \) in the intermediate sector:

For cylindrical gears: \( 1.15 < N < 1.5 \)

For bevel gears: \( 1.25 < N < 1.5 \)

Comments raised in 1.8.2.4 and 1.8.2.5 should be observed.

\( K_v \) is determined by linear interpolation between \( K_v \) for \( N = 1.15 \) resp. 1.25 and \( N = 1.5 \) as

1.8.3 Multi-resonance method

For high speed gear \( (v > 40 \text{ m/s}) \), for multmesh medium speed gears, for gears with significant lateral shaft flexibility etc. it is advised to determine \( K_v \) on basis of relevant dynamic analysis.

Incorporating lateral shaft compliance requires transformation of even a simple pinion-wheel system into a multi-mass system. It is advised to incorporate all relevant inertias and torsional shaft stiessnesses into an equivalent (to pinion speed) system. Thereby the mesh stiffness appears as an equivalent torsional stiffness.
The natural frequencies are found by solving the set of differential equations (one equation per inertia mass). Note that for a gear put on a laterally flexible shaft, the coupling bending-torsionals is arranged by introducing the gear mass and the lateral stiffness with its relation to the torsional displacement and torque in that shaft.

Only the natural frequency (ies) having high relative displacement (or torque) through the actual pinion-wheel flexible element, need(s) to be considered as critical frequency (ies).

The level of the dynamic factor may also be determined on basis of simulation technique using numeric integration with relevant tooth stiffness variation and pitch/profile errors.

### 1.9 Face Load Factors, $K_{H\beta}$ and $K_{F\beta}$

The face load factors, $K_{H\beta}$ for contact stresses and for scuffing, $K_{F\beta}$ for tooth root stresses, account for non-uniform load distribution across the facewidth.

$K_{H\beta}$ is defined as the ratio between the maximum load per unit facewidth and the mean load per unit facewidth.

$K_{F\beta}$ is defined as the ratio between the maximum tooth root stress per unit facewidth and the mean tooth root stress per unit facewidth. The mean tooth root stress relates to the considered facewidth $b_1$ resp. $b_2$.

Note that facewidth in this context is the design facewidth $b$, even if the ends are unloaded as often applies to e.g. bevel gears.

The plane of contact is considered.

#### 1.9.1 Relations between $K_{H\beta}$ and $K_{F\beta}$

**Cylindrical gears**

$$K_{F\beta} = K_{H\beta} \frac{1}{1 + \frac{h}{b_1} + \frac{b_2}{(h/b)^2}}$$

where $h/b$ is the ratio tooth height/facewidth. The maximum of $b_1/b_1$ and $b_2/b_2$ is to be used. For double helical gears, use only the facewidth of one helix.

This relation between $K_{F\beta}$ and $K_{H\beta}$ is only valid for gears with the hardest contact towards a tooth end and $b_1 < b_2 < b$.

For gears with end relief or crowning, where the ends are lightly loaded or even unloaded, the following applies:

$$K_{F\beta} = K_{H\beta}$$

If the tooth root facewidth ($b_1$ or $b_2$) is considerably wider than $b$, the value of $K_{F\beta} / K_{H\beta}$ may even exceed $K_{H\beta}$.

**Bevel gears**

$$K_{F\beta} = \frac{K_{H\beta}}{K_{F\beta}}$$

$K_{F\beta} = 0.211 \left( \frac{c_{m}}{R_m} \right)^q + 0.789$ for spiral bevel gears.

$K_{F\beta} = 1$ for other bevel gears.

where:

- $c_{m} = $ cutter radius.
- $R_m = $ mean cone distance.
- $q = \log (\sin \beta_m)$

If $K_{F\beta}$ calculated $< 1$, use $K_{F\beta} = 1$.

If $K_{F\beta}$ calculated $> 1.15$, use $K_{F\beta} = 1.15$.

#### 1.9.2 Measurement of face load factors

Primarily, $K_{F\beta}$ may be determined by a number of strain gauges distributed over the facewidth. Such strain gauges must be put in exactly the same position relative to the root fillet. Relations in 1.9.1 apply for conversion to $K_{F\beta}$.

Secondarily, $K_{H\beta}$ may be evaluated by observed contact patterns on various defined load levels. It is imperative that the various test loads are well defined. Usually, it is also necessary to evaluate the elastic deflections. Some teeth at each 90 degrees are to be painted with a suitable lacquer. Always consider the poorest of the contact patterns.

After having run the gear for a suitable time at test load 1 (the lowest), observe the contact pattern with respect to expansion over the facewidth. Evaluate that $K_{H\beta}$ by means of the methods mentioned in this Section. Proceed in the same way for the next higher test load etc., until there is a full face contact pattern. From these data, the initial mesh misalignment (i.e. without elastic deflections) can be found by extrapolation, and then also the $K_{H\beta}$ at design load can be found by calculation and extrapolation. See example.

![Example on experimental determination of $K_{H\beta}$](Fig. 1.1)

It must be considered that inaccurate gears may accumulate a larger observed contact pattern than the actual single mesh to mesh contact patterns. This is particularly important for lapped bevel gears. Ground or hard metal hobbed bevel gears are assumed to present an accumulated contact pattern which is practically equal the actual single mesh to mesh contact pattern. As a rough guidance, the (observed) accumulated contact pattern for lapped bevel gears may be reduced by 10% in order to assess the single mesh to mesh contact pattern which is used in 1.9.9.
1.9.3 Theoretical determination of \( K_{1B} \)

The methods described in 1.9.3 to 1.9.8 may be used for cylindrical gears. The principles may to some extent also be used for bevel gears, but a more practical approach is given in 1.9.9.

General: For gears where the tooth contact pattern cannot be verified during assembly or under load, all assumptions are to be well on the safe side.

\( K_{1B} \) is to be determined in the plane of contact.

The influence parameters considered in this method are:

- mean mesh stiffness \( c_y \) (see 1.11) (if necessary, also variable stiffness over \( b \))
- mean unit load \( F_{mn}/b = F_{mn} K_A K_v K_w/\gamma \)
- misalignment \( f_m \) due to deflections of shafts and gear bodies (both pinion and wheel)
- misalignment \( f_{el} \) due to deflections of bearings
- misalignment \( f_c \) due to bearing clearances and tolerances
- misalignment \( f_s \) due to manufacturing tolerances
- helix modifications as crowning, end relief, helix correction
- running in amount \( y_B \) (see 1.12)

In practice several other parameters such as centrifugal expansion, thermal expansion, housing deflection, etc. contribute to \( K_{1B} \). However, these parameters are not taken into account unless in special cases when being considered as particularly important.

When all or most of the a.m. parameters are to be considered, the most practical way to determine \( K_{1B} \) is by means of a graphical approach, described in 1.9.3.1.

If \( c_y \) can be considered constant over the facewidth, and no helix modifications apply, \( K_{1B} \) can be determined analytically as described in 1.9.3.2.

1.9.3.1 Graphical method

The graphical method utilizes the superposition principle, and is as follows:

- Calculate the mean mesh deflection
  \[ \delta_M = F_{mn}/(b c_y) \] (\( c_y \) is the average mesh stiffness over the facewidth \( b \))
- Draw a base line with length \( b \), and draw up a rectangular with height \( \delta_M \). (The area \( \delta_M b \) is proportional to the transmitted force)
- Calculate the elastic deflection \( f_{sh} \) in the plane of contact. Balance this deflection curve around a zero line, so that the areas above and below this zero line are equal.

\[ \text{Fig. 1.2} \]

\( f_{sh} \) balanced around a zero line.

Superimpose these ordinates of the \( f_{sh} \) curve to the previous load distribution curve. (The area under this new load distribution curve is still \( \delta_M b \))

- Calculate the influence of the bearing deflections \( f_{be} \) in the plane of contact. This is a straight line and is balanced around a zero line as indicated in Fig. 1.3, but with one distinct direction. Superimpose these ordinates to the previous load distribution curve.
- The amount of crowning, end relief or helix correction (defined in the plane of contact) is to be balanced around a zero line similarly to \( f_{sh} \).

\[ \text{Fig. 1.3} \]
Crowning \( C_c \) balanced around a zero line.

Superimpose these ordinates to the previous load distribution curve. In case of high crowning etc., as e.g. often applied to bevel gears, the new load distribution curve may cross the base line (the real zero line). The result is areas with negative load which is not real, as the load in these areas should be zero. Thus corrective actions must be made, but for practical reasons it may be postponed to after next operation.

- The amount of initial mesh misalignment, \( f_{im} + f_{ic} \) (defined in the plane of contact), is to be balanced around a zero line. If the direction of \( f_{im} + f_{ic} \) is known (due to initial contact check), or if the direction of \( f_{ic} \) is known due to design (e.g. overhang bevel pinion), this should be taken into account. If direction unknown, the influence of \( f_{im} + f_{ic} \) in both directions as well as equal zero, should be considered.

\[ \text{Fig. 1.4} \]
\( f_{im} + f_{ic} \) in both directions, balanced around a zero line.

Superimpose these ordinates to the previous load distribution curve. This results in up to 3 different curves, of which the one with the highest peak is to be chosen for further evaluation.

- If the chosen load distribution curve crosses the base line (i.e. mathematically negative load), the curve is to be corrected by adding the negative areas and dividing this with the active facewidth. The (constant) ordinates of this rectangular correction area, are to be subtracted from the positive part of the load distribution curve.
It is advisable to check that the area covered under this new load distribution curve is still equal $\delta M$. b.

- If $c_0$ cannot be considered as constant over b, then correct the ordinates of the load distribution curve with the local (on various positions over the facewidth) ratio between local mesh stiffness and average mesh stiffness $c_1$ (average over the active facewidth only).

Note that the result is to be a curve which covers the same area $\delta M$ as before.

- The influence of running in $y_p$ is to be determined as in 1.12 whereby the value for $F_{px}$ is to be taken as twice the distance between the peak of the load distribution curve and $\delta M$.

- Determine

$$K_{HP} = \frac{\text{peak of curve} \pm 0.5 y_p}{\delta_M}$$

1.9.3.2 Simplified analytical method

The analytical approach is similar to 1.9.3.1 but has a more limited application as $c_0$ is assumed constant over the facewidth and no helix modification applies.

- Calculate the elastic deflection $f_{sh}$ in the plane of contact. Balance this deflection curve around a zero line, so that the area above and below this zero line are equal, see Fig. 1.2.

The max. positive ordinate is $1/2 \Delta f_{sh}$.

- Calculate the initial mesh alignment as

$$F_{px} = |\Delta f_{sh} + f_{ma} \pm f_{sh} \pm f_{del}|$$

The negative signs may only be used if this is justified and/or verified by a contact pattern test. Otherwise, always use positive signs. If a negative sign is justified, the value of $F_{px}$ is not to be taken less than the largest of each of these elements.

- Calculate the effective mesh misalignment as

$$F_{p} = F_{px} - y_p$$

$(y_p$ see 1.12)

- Determine

$$K_{HP} = l + \frac{c_f F_{p} b}{2 F_m} \quad \text{for} \quad K_{HP} \leq 2$$

or

$$K_{HP} = \frac{2 c_f F_p b}{F_m} \quad \text{for} \quad K_{HP} > 2$$

1.9.4 Determination of $f_{sh}$

$f_{sh}$ is the mesh misalignment due to elastic deflections. Usually it is sufficient to consider the combined mesh deflection of the pinion body and shaft and the wheel shaft. The calculation is to be made in the plane of contact (of the considered gear mesh), and to consider all forces (incl. axial) acting on the shafts. Forces from other meshes can be parted into components parallel resp. vertical to the considered plane of contact. Forces vertical to this plane of contact have no influence on $f_{sh}$.

It is advised to use the diameter $d + 2 x m_b$ for bending and shear deflection of a toothed element, and diameter $d + 2 x m_b [x = b_{sh0} + 0.2]$ for torsional deflection of a toothed element.

Usually, $f_{sh}$ is calculated on basis of an evenly distributed load. If the analysis of $K_{HP}$ shows a considerable misdistribution in term of hard end contact, or if it is known by other reasons that there exists a hard end contact, the load should be correspondingly distributed when calculating $f_{sh}$. In fact, the whole $K_{HP}$ procedure can be used iteratively. 2-3 iterations will be enough, even for almost triangular load distributions.

1.9.5 Determination of $f_{del}$

$f_{del}$ is the mesh misalignment due to bearing deflections (housing deflection may be included if determined). There is a principle difference in the approach for rolling bearings and journal bearings.

For rolling bearings $f_{del}$ is determined on basis of the elastic deflection of the bearings in the plane of contact. An elastic bearing deflection depends on the bearing load and size and number of rolling elements. Note that the bearing clearances and tolerances are not included here. $f_{del}$ is the facewidth times the angular misalignment due to different bearing deflections, both pinion and wheel shaft to be considered.

For journal bearings $f_{del}$ is determined on basis of the lift of the shafts due to lubrication oil film thickness. Note that $f_{del}$ takes into account the influence of the bearing clearances and tolerances. $f_{del}$ is the facewidth times the angular misalignment due to different oil film thicknesses in the bearings and/or due to different journal positions in the bearings caused by other forces than the forces from the considered gear mesh.

1.9.6 Determination of $f_{ma}$

$f_{ma}$ is the mesh misalignment due to bearing clearances. In principle $f_{ma}$ and $f_{del}$ could be combined. But as $f_{del}$ can be determined by analysis and has a distinct direction, and $f_{ma}$ is to a large extent dependent on tolerances and in many cases has no distinct direction (i.e. ± tolerance), it is practicable to separate these two influences.

Due to different bearing clearances (incl. tolerances) in both pinion and wheel shafts the shaft axis will have an angular misalignment in the plane of contact. $f_{ma}$ is the facewidth times this angular misalignment. Note that $f_{ma}$ may have a distinct direction or be given as a ± tolerance, or a combination of both.

$f_{ma}$ is particularly important for overhang designs, for gears with widely different kinds of bearings on each side, and when the bearings have wide tolerances on clearances. In general it shall be possible to replace standard bearings without causing the real load distribution to exceed the design premises. For high speed gears with journal bearings, the expected wear should also be considered.

1.9.7 Determination of $f_{mg}$

$f_{mg}$ is the mesh misalignment due to manufacturing errors of pinion, wheel and housing bore. Referring to ISO 1328-1975, $f_{mg}$ may be taken according to the poorest grade of accuracy for pinion, wheel and housing bore (i.e. taken equal to 1 times the poorest of these 3 elements, not adding the tolerances of these 3). The above mentioned ISO standard is only valid up to $b = 160$ mm. Beyond this, the tolerances have to be given on the drawings.

1.9.8 Comments to various gear types

For double helical gears, $K_{hp}$ is to be determined for both helices. Usually an even load share between the helices can be assumed.

For planetary gears the sun pinion suffers only twist, no bending. It must be noted that the total twist is the sum of
the twist due to each mesh. If the value of $K_T \neq 1$, this must be taken into account when calculating the total sun pinion twist (i.e. twist calculated with the force per mesh without $K_T$, and multiplied with the number of planets).

When planets are mounted on spherical bearings, the mesh misalignments sun—planet resp. planet—annulus will be balanced, i.e. the misalignment will be the average between the two theoretical individual misalignments. The face load distribution on the flanks of the planets can take full advantage of this. However, as the sun and annulus mesh with several planets with possibly widely different lead errors, the sun and annulus cannot obtain the above mentioned advantage to the full extent.

1.9.9 Determination of $K_{HB}$ for bevel gears

If a theoretical approach similar to 1.9.3—1.9.8 is not documented, the following may be used:

$$K_{HB} = 1.5 \frac{h_{cH}/b}{K_{HB} \beta}$$

$h_{cH}/b$ represents the relative active facewidth.

Higher values than $h_{cH}/b = 0.9$ are not to be used.

$K_{HB} \beta$ represents the influence of the bearing arrangement on the face load distribution.

If the contact pattern will be verified (see 1.9.2) at rated load, the specified minimum $h_{cH}/b$ may be used in the above equation and $K_{HB} \beta$ be put unity.

If the contact pattern is only checked at light or no load, the expected (due to experience with similar gears) $h_{cH}/b$ ratio may be used, and $K_{HB} \beta$ as follows:

<table>
<thead>
<tr>
<th>$K_{HB} \beta$</th>
<th>Neither pinion nor wheel overhanging mounted</th>
<th>Only wheel overhanging mounted</th>
<th>Only pinion overhanging mounted</th>
<th>Both pinion and wheel overhanging mounted</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.10</td>
<td>1.15</td>
<td>1.20</td>
<td>1.30</td>
<td></td>
</tr>
</tbody>
</table>

1.10 Transversal Load Distribution Factors, $K_{HB}$ and $K_{Fa}$

The transversal load distribution factors, $K_{HB}$ for contact stresses and for scuffing, $K_{Fa}$ for tooth root stresses account for the effects of pitch and profile errors on the transversal load distribution between 2 or more pairs of teeth in mesh.

The following relations may be used:

$$K_{Fa} = K_{HB} = \frac{c_p}{2} \left( 0.9 + 0.4 \frac{c_p (f_{ph} - y_a) b}{F_{IH}} \right)$$

valid for $c_p \leq 2$

$$K_{Fa} = K_{HB} = 0.9 + 0.4 \sqrt{2 (c_p - 1) \frac{c_p (f_{ph} - y_a) b}{F_{IH}}}$$

valid for $c_p > 2$

where:

$F_{IH} = F_1 K_A K_T K_C K_{HB}$

$c_p = \text{See 1.11}$

$y_a = \text{See 1.12}$

$f_{ph} = \text{Maximum adjacent base pitch error (um) of pinion or wheel, or maximum profile error of pinion or wheel if this is larger than the max. adjacent pitch error.}$

Note: In case of adequate tip relief adapted to the load, half of the above mentioned $f_{ph}$ can be introduced.

A tip relief is considered adequate when it is within $\pm 30\%$ of the value in 4.3.2:

Limitations of $K_{Fa}$ and $K_{Fa}$:

If the calculated values for

$K_{Fa} = K_{HB} < 1$, use $K_{Fa} = K_{HB} = 1.0$

If the calculated value of

$K_{Fa} > \frac{c_p}{c_p z_{a1}^{2}}$, use $K_{Fa} = \frac{c_p}{c_p z_{a1}^{2}}$

If the calculated value of $K_{Fa} > \frac{c_p}{c_p z_{a1}^{2}}$, use $K_{Fa} = \frac{c_p}{c_p z_{a1}^{2}}$

1.11 Tooth Stiffness Constants, $c'$ and $c_p$

The tooth stiffness constant is defined as the load which is necessary to deform one or several meshing gear teeth having 1 mm facewidth by an amount of 1 µm, in the plane of contact.

$c'$ is the maximum stiffness of a single pair of teeth.

$c_p$ is the mean value of the mesh stiffness in a transverse plane (brief term: mesh stiffness).

Cylindrical gears

The following relation may be used for gears with reasonably high unit load $F_1 K_A K_{HB}$:

$$c' = 0.8 \cos \beta \frac{C_{R/C_{BS}}}{q}$$

and

$$c_p = c'(0.75 \varepsilon_q + 0.25)$$

where:

$$C_{BS} = \frac{1 + 0.5 \left(1.2 - \frac{h_{d1} + h_{d0}}{2}ight)}{1 - 0.02 (20 - \sigma_h)}$$

$$q_1 = 0.04723 + 0.15551 x_1 \frac{z_{a1}}{z_{a2}} + 0.25791 x_1 \frac{z_{a1}}{z_{a2}} - 0.00635 x_1 - 0.11654 x_1 \frac{z_{a1}}{z_{a2}} - 0.00193 x_2 - 0.24188 x_2 + 0.0529 x_1^2 + 0.0182 x_2^2$$

(for internal gears, use $z_{a2}$ equal infinite and $x_2 = 0$).

$h_{d0} = h_{d0}$ for all practical purposes.

$C_{R}$ considers the increased flexibility of the wheel teeth if the wheel is not a solid disc, and may be calculated as:

$$C_R = 1 + \frac{\ln \left( \frac{h_{d1}}{h_{d0}} \right)}{S_C (t_R / S_m)}$$

where:

**DET NORSKE VERITAS**
b_H = thickness of a central web
s_R = average thickness of rim

The formula is valid for \( b_H / b \geq 0.2 \) and \( s_R / t_n \geq 1 \). Outside this range of validity and if the web is not centrally positioned, \( C_R \) has to be specially considered.

**Note:**

\( C_R \) is the ratio between the average mesh stiffness over the face-width and the mesh stiffness of a gear pair of solid discs. The local mesh stiffness in way of the web corresponds to the mesh stiffness with \( C_R = 1 \). The local mesh stiffness where there is no web support will be less than calculated with \( C_R \) above. Thus, e.g., a centrally positioned web will have an effect corresponding to a longitudinal crowning of the teeth. See also 1.9 regarding \( K_{HP} \).

**Note:**

For a simplified approach the following may be used:

\[ c_y = 20 \]

If the a.m. unit load for cylindrical gears is less than 100 N/mm, the stiffness can be decreased linearly with actual unit load. The influence of this regarding the natural frequency \( n_p \) (see 1.8, \( K_v \)) should be considered.

**Bevel gears**

In lack of more detailed relationship between stiffness and geometry the following may be used:

\[ c' = 14 \; \frac{b_H}{0.85 \; b} \; \quad c_y = 20 \; \frac{b_H}{0.85 \; b} \]

\( b_H \) not to be used in excess of 0.85 \( b \) in these formulae.

Bevel gears with heightwise and lengthwise crowning have a progressive mesh stiffness. The values mentioned above are only valid for high loads. They should not be used for determination of \( C_{eff} \) (see 4.3.2).

Further, when extrapolating part load contact patterns (for accurate gears) into expected full load contact pattern, the stiffness progressivity has to be considered.

With linear stiffness \( b_H / b \sim \text{load}^{1/3} \), but in practice the exponent is 1/4 to 1/5.

### 1.12 Running-in Allowances

The running-in allowances account for the influence of running-in wear on the various error elements.

\( y_a \) resp. \( y_b \) are the running-in amounts which reduce the influence of pitch and profile errors, resp. influence of localized faceload.

\( C_{ay} \) is defined as the running-in amount which compensates for lack of tip relief.

The following relations may be used:

For not surface hardened steel

\[ y_a = \frac{160}{\sigma_{Hlim}} \; f_{pb} \]

\[ y_b = \frac{320}{\sigma_{Hlim}} \; F_{bx} \]

with the following upper limits:

<table>
<thead>
<tr>
<th>( \nu )</th>
<th>( \leq 5 ; \text{m/s} )</th>
<th>( 5-10 ; \text{m/s} )</th>
<th>( &gt;10 ; \text{m/s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_a ) max</td>
<td>none</td>
<td>( \frac{12800}{\sigma_{Hlim}} )</td>
<td>( \frac{6400}{\sigma_{Hlim}} )</td>
</tr>
<tr>
<td>( y_b ) max</td>
<td>none</td>
<td>( \frac{25600}{\sigma_{Hlim}} )</td>
<td>( \frac{12800}{\sigma_{Hlim}} )</td>
</tr>
</tbody>
</table>

For surface hardened steel

\( y_a = 0.075 \; f_{pb} \) but not more than 3 for any speed

\( y_b = 0.15 \; F_{bx} \) but not more than 6 for any speed

For all kinds of steel

\[ C_{ay} = \frac{1}{18} \; \left( \frac{\sigma_{Hlim}}{97} - 18.45 \right)^2 + 1.5 \]

When pinion and wheel material differ, the following applies:

- Use the larger of \( f_{pb} - y_a \) and \( f_{pb} - y_a \) to replace \( f_{pb} - y_a \) in the calculation of \( K_{HP} \).
- Use \( y_b = \frac{1}{2} \; (y_{b1} + y_{b2}) \) in the calculation of \( K_{HP} \).
- Use \( C_a = \frac{1}{2} \; (C_{ay1} + C_{ay2}) \) in the calculation of \( K_v \).
- Use \( C_{a1} = C_{a2} = \frac{1}{2} \; (C_{ay1} + C_{ay2}) \) in the scuffing calculation if no design tip relief is foreseen.
2. Calculation of Surface Durability

2.1 Scope and General Remarks

Part 2 includes the calculation of flank surface durability as limited by pitting, spalling, case crushing and subsurface yielding. Endurance and time limited flank surface fatigue is calculated by means of 2.2—2.13.

Pitting itself is not considered as a critical damage for slow speed gears. However, pitting can create a severe notch effect which may result in tooth breakage. This is particularly important for surface hardened teeth, but also for high strength through hardened teeth. For high speed gears, pitting is not permitted.

Spalling and case crushing are considered similar to pitting, but may have a more severe effect on tooth breakage due to the larger material breakouts, initiated below the surface.

For jacking gears (self elevating offshore units) or similar slow speed gears designed for very limited life, the maximum (or very slow running) surface load for surface hardened flanks is limited by the subsurface yield strength.

For case hardened gears operating with relatively thin lubrication oil films, gray staining (micropitting) may be the limiting criterion for the gear rating. Specific calculation methods for this purpose are not given here, but are under consideration for future revisions. Thus depending on experience with similar gear designs, limitations on surface durability rating other than those acc. to 2.2—2.13 may be applied.

2.2 Basic Equations

Calculation of surface durability for spur gears and straight bevel gears is based on the contact stress at the inner point of single pair contact or the contact stress at the pitch point, whichever is greater.

Calculation of surface durability for helical gears is based on the contact stress at the pitch point.

Calculation of surface durability for spiral bevel gears (\( \eta_p > 1 \)) is based on the contact stress at the midpoint of the zone of contact.

For gears with \( 0 < \eta_p < 1 \), a linear interpolation between the above mentioned applies.

The contact stress \( \sigma_H \) and the permissible contact stress \( \sigma_{HP} \) are to be calculated for both pinion and wheel.

Contact stress:

**Cylindrical gears**

\[
\sigma_H = Z_{B,D} Z_H Z_E Z_b \sqrt{\frac{F_t (u + 1)}{d_1 b u}} K
\]

where:

\[
K = K_A K_{E} K_v K_{HP} K_{HZ}
\]

\( Z_{H,D} \) = Zone factor for inner point of single pair contact for pinion resp. wheel (see 2.3).

\( Z_H \) = Zone factor for pitch point (see 2.3).

\( E \) = Elasticity factor (see 2.4).

\( Z_v \) = Contact ratio factor (see 2.5).

\( Z_b \) = Helix angle factor (see 2.7).

\( F_t, K_A, K_v, K_{HP}, K_{HZ} \) see 1.5—1.10.

\( d_1, b, u \) see 1.2—1.5.

**Bevel gears**

\[
\sigma_H = Z_{M-B} Z_H Z_E Z_{LS} Z_b Z_K \sqrt{\frac{F_{mt} - w_u + 1}{d_{v1} b_u}} K
\]

where:

\( Z_H, Z_6, Z_b, K \) see above.

\( Z_{M-B} \) = mid-zone factor, see 2.3.

\( Z_{LS} \) = load sharing factor, see 2.6.

\( Z_K \) = bevel gear factor, see 2.8.

\( F_{mt}, w_u, b_u, h_0 \) see 1.2—1.5.

The application of these factors is only valid when the heightwise crowning is chosen so that the contact zone just spreads to the tooth tips of pinion and wheel under rated load.

If the contact zone ends below the tooth tip of either pinion and/or wheel, the contact stress is to be multiplied with

\[
\sqrt{\frac{m_{hm}}{m_{hm} - \Delta}}
\]

where \( \Delta \) is the larger of the distance (mm) between contact zone and tooth tip of pinion and wheel.

If the contact zone reaches the tooth tip of either pinion and/or wheel over a width of 0,25 b or more, special attention must be given to scuffing and gray staining.

Permissible contact stress:

\[
\sigma_{HP} = \frac{\sigma_{Hlim} Z_N Z_L Z_v Z_R Z_W Z_X}{S_H}
\]

where:

\( \sigma_{Hlim} \) = Endurance limit for contact stresses (see 2.9).

\( Z_N \) = Life factor for contact stresses (see 2.10).

\( S_H \) = Required safety factor acc. to the rules.

\( Z_L, Z_v, Z_R \) = Oil film influence factors (see 2.11).

\( Z_W \) = Work hardening factor (see 2.12).

\( Z_X \) = Size factor (see 2.13).

2.3 Zone Factors \( Z_H, Z_{B,D} \) and \( Z_{M-B} \)

The zone factor, \( Z_H \), accounts for the influence on contact stresses of the tooth flank curvature at the pitch point and converts the tangential force at the reference cylinder to the normal force at the pitch cylinder.

**Cylindrical gears**

\[
Z_H = \frac{2 \cos \beta_H \cos \alpha_{sw}}{\cos^2 \phi_{sw} \sin 2 \alpha_{sw}}
\]

**Bevel gears**

\[
Z_H = \frac{2 \cos \beta_{bm}}{\sin (2 \alpha_{v})}
\]
The zone factors, $Z_B$, account for the influence on contact stresses of the tooth flank curvature at the inner point of single pair contact in relation to $Z_H$. Index $B$ refers to pinion, $D$ to wheel.

**Cylindrical gears**

For $\epsilon_\beta \geq 1$, $Z_{B,D} = 1$

For $\epsilon_\beta = 0$ (spur gears)

$$Z_B = \frac{\tan \alpha_{tw}}{\sqrt{\left(\frac{F_{01}}{F_{11}} - \frac{F_{11}}{\pi z_1}\right)\left(\frac{F_{02}}{F_{22}} - \frac{F_{22}}{\pi z_2}\right)}}$$

$$Z_D = \frac{\tan \alpha_{lw}}{\sqrt{\left(\frac{F_{02}}{F_{22}} - \frac{F_{22}}{\pi z_2}\right)\left(\frac{F_{01}}{F_{11}} - \frac{F_{11}}{\pi z_1}\right)}}$$

where

$$F_{01} = \sqrt{\left(\frac{d_{z1}}{d_{b1}}\right)^2 - 1}$$

$$F_{02} = \sqrt{\left(\frac{d_{z2}}{d_{b2}}\right)^2 - 1}$$

$$F_1 = 2$$

$$F_2 = 2(\epsilon_a - 1)$$

For $0 < \epsilon_\beta < 1$

$$Z_{B,D} = 1 + (1 - \epsilon_\beta)(Z_{B,D} \text{ (for spur gears)} - 1)$$

**Bevel gears**

The mid-zone factor transforms $Z_H$ and thereby the contact stress at the pitch point to that at the determinant point of load application.

$$Z_{M-B} = \frac{\tan \alpha_{vl}}{\sqrt{\left(F_{v01} - \frac{F_{11}}{Z_{h1}}\right)\left(F_{v02} - \frac{F_{22}}{Z_{h2}}\right)}}$$

where

<table>
<thead>
<tr>
<th>$\epsilon_\beta$</th>
<th>$F_1$</th>
<th>$F_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_\beta = 0$</td>
<td>$2$</td>
<td>$2(\epsilon_a - 1)$</td>
</tr>
<tr>
<td>$0 &lt; \epsilon_\beta &lt; 1$</td>
<td>$2 + (\epsilon_a - 2)\epsilon_\beta$</td>
<td>$2\epsilon_a - 2 + (2 - \epsilon_\beta)\epsilon_\beta$</td>
</tr>
<tr>
<td>$\epsilon_\beta \geq 1$</td>
<td>$\epsilon_a$</td>
<td>$\epsilon_a$</td>
</tr>
</tbody>
</table>

$$F_{v01} = \sqrt{\left(\frac{d_{val}}{d_{bvl}}\right)^2 - 1}$$

$$F_{v02} = \sqrt{\left(\frac{d_{v2}}{d_{b2}}\right)^2 - 1}$$

**Note:**

For cylindrical or bevel gears with very low number of teeth the inner contact point (A) may be close to the base circle. In order to avoid a wear edge near A, it is required to have suitable tip relief on the wheel.

### 2.4 Elasticity Factor, $Z_E$

The elasticity factor, $Z_E$, accounts for the influence of the material properties as modulus of elasticity and Poisson's ratio on the contact stresses.

For steel against steel $Z_E = 189.8$

### 2.5 Contact Ratio Factor, $Z_k$

The contact ratio factor $Z_k$ accounts for the influence of the transverse contact ratio $\epsilon_a$ and the overlap ratio $\epsilon_\beta$ on the contact stresses.

$$Z_k = \frac{4 - \epsilon_a}{3} - (1 - \epsilon_a) + \frac{\epsilon_\beta}{\epsilon_a} \text{ for } \epsilon_\beta < 1$$

$$Z_k = \sqrt{\frac{1}{\epsilon_\beta}} \text{ for } \epsilon_\beta \geq 1$$

### 2.6 Load Sharing Factor, $Z_{LS}$

The load sharing factor $Z_{LS}$ accounts for load sharing between two or more pair of teeth in contact for $\epsilon_\gamma > 2$.

The load distribution across a contact line is assumed to be of elliptical shape. The distribution of peak loads of the contact lines is assumed to be a parabola with exponent 1.5.

For $\epsilon_\gamma < 2$,

$$Z_{LS} = 1$$

(In practice, when $\epsilon_\beta < 1$ also $\epsilon_\gamma < 2$.)

For $\epsilon_\gamma > 2$ and $\epsilon_\beta > 1$

$$Z_{LS} = \left(1 + 2 + \left(\frac{2}{\epsilon_\gamma}\right)^{1.5}\right)^{-1/2}$$

If $Z_{LS}$ calculated $< \sqrt{0.7}$, then $Z_{LS} = \sqrt{0.7}$

### 2.7 Helix Angle Factor, $Z_\beta$

The helix angle factor, $Z_\beta$, accounts for the influence of helix angle (independent of its influence on $Z_a$) on the surface durability.

**Cylindrical gears**

$$Z_\beta = \sqrt{\cos \beta}$$

**Bevel gears**

$$Z_\beta = \sqrt{\cos \beta_m}$$

### 2.8 Bevel Gear Factor, $Z_K$

The bevel gear factor accounts for the difference between bevel and cylindrical gear loading and adjusts the contact stresses in such a way that the same permissible stresses may apply.

The following may be used:

$$Z_K = 0.8$$
2.9 Values of Endurance Limit, $\sigma_{Him}$ and Static Strengths, $\sigma_{H10^5}$, $\sigma_{H10^7}$

$\sigma_{Him}$ is the limit of contact stress that may be sustained for $5 \times 10^7$ cycles, without the occurrence of progressive pitting. For most materials $5 \times 10^7$ cycles are considered to be the beginning of the endurance strength range or lower knee of the of the $\sigma$–$N$ curve. (see also Life Factor $Z_N$). However, for nitried steels $2 \times 10^6$ applies.

For this purpose, pitting is defined by

- for not surface hardened gears: pitted area $\geq 2\%$ of total active flank area
- for surface hardened gears: pitted area $\geq 0.5\%$ of total active flank area, or $\geq 4\%$ of one particular tooth flank area

$\sigma_{H10^5}$ and $\sigma_{H10^7}$ are the contact stresses which the given material can withstand for $10^5$ resp. $10^7$ cycles without subsurface yielding or flank damages as pitting, spalling or case crushing.

The following listed values for $\sigma_{Him}$, $\sigma_{H10^5}$ and $\sigma_{H10^7}$ may only be used for materials subjected to a quality control as the one referred to in the rules.

Results of approved fatigue tests may also be used as the basis for establishing these values.

The defined survival probability is 99%.

<table>
<thead>
<tr>
<th>Alloyed case hardened steels (surface hardness 56–63 HRC):</th>
<th>$\sigma_{Him}$</th>
<th>$\sigma_{H10^5}$</th>
<th>$\sigma_{H10^7}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>— of specially approved high grade. — of normal grade.</td>
<td>1650</td>
<td>2500</td>
<td>3100</td>
</tr>
<tr>
<td>Nitriding steel of approved grade; gas nitrided (surface hardness 700–800 HV):</td>
<td>1500</td>
<td>2400</td>
<td>3100</td>
</tr>
<tr>
<td>Alloyed quenched and tempered steel, bath or gas nitrided (surface hardness 500–700 HV):</td>
<td>1250</td>
<td>1.3 $\sigma_{Him}$</td>
<td>1.3 $\sigma_{Him}$</td>
</tr>
<tr>
<td>Alloyed, flame or induction hardened steel (surface hardness 500–650 HV):</td>
<td>1000</td>
<td>1.3 $\sigma_{Him}$</td>
<td>1.3 $\sigma_{Him}$</td>
</tr>
<tr>
<td>Alloyed quenched and tempered steel:</td>
<td>0.75 HV + 750</td>
<td>1.6 $\sigma_{Him}$</td>
<td>4.5 HV</td>
</tr>
<tr>
<td>Carbon steel:</td>
<td>1.4 HV + 350</td>
<td>1.6 $\sigma_{Him}$</td>
<td>4.5 HV</td>
</tr>
<tr>
<td></td>
<td>1.5 HV + 250</td>
<td>1.6 $\sigma_{Him}$</td>
<td>4.5 HV</td>
</tr>
</tbody>
</table>

These values refer to forged or hot rolled steel. For cold steel the values for $\sigma_{Him}$ are to be reduced by 15%.

2.10 Life Factor, $Z_N$

The life factor, $Z_N$, takes account of a higher permissible contact stress if only limited life (number of cycles, $N_L$) is demanded or lower permissible contact stress if very high number of cycles apply.

If this is not documented by approved fatigue tests, the following method may be used:

For all steels except nitried:

$N_L \geq 5 \times 10^7$: $Z_N = 1$ or $Z_N = \left( \frac{5 \times 10^7}{N_L} \right)^{0.0307}$

The $Z_N = 1$ from $5 \times 10^7$ on may only be used when optimum lubrication and high material cleanliness applies.

$10^3 < N_L < 5 \times 10^7$ : $Z_N = \left( \frac{5 \times 10^7}{N_L} \right)^{0.17 \log Z_{NH}}$

$N_L = 10^3$ : $N_L = Z_{NH} \sigma_{Him} = \frac{\sigma_{Him} Z_{L} Z_{R} Z_{X}}{\sigma_{Him} Z_{L} Z_{R} Z_{X}}$

For nitried steels:

$Z_L Z_{R} Z_{X} < 2 \times 10^6$: $Z_N = 1$ or $Z_N = \left( \frac{2 \times 10^6}{N_L} \right)^{0.0191}$

The $Z_N = 1$ from $2 \times 10^6$ on may only be used when optimum lubrication and high material cleanliness applies.

$10^5 < N_L < 2 \times 10^6$ : $Z_N = \frac{2 \times 10^6}{N_L}^{0.7538 \log Z_{NH}}$

$N_L \leq 10^5$ : $Z_N = Z_{NB} = \frac{1.3}{Z_L Z_{R} Z_{X}}$

* Note that when no index indicating number of cycles is used, the factors are valid for $5 \times 10^7$ (resp. $2 \times 10^6$ for nitried) cycles.

2.11 Influence Factors on Lubrication Film, $Z_L$, $Z_V$ and $Z_K$

The lubricant factor, $Z_L$, accounts for the influence of the type of lubricant and its viscosity, the speed factor, $Z_V$, accounts for the influence of the pitch line velocity and the roughness factor, $Z_K$, accounts for influence of the surface roughness on the surface endurance capacity.

The following methods may be applied in connection with the endurance limit:
Classification Notes — No. 41.2

Surface hardened steels | Not surface hardened steels
---|---

\[ Z_L = 0.91 + \frac{0.36}{(1.2 + 134/\nu_0)^2} \]

\[ Z_V = 0.93 + \frac{0.14}{\sqrt{0.8 + (32/\nu)}} \]

\[ Z_R = \left( \frac{3}{R_{Z_{rel}}} \right)^{0.08} \]

\[ Z_L = 0.83 + \frac{0.68}{(1.2 + 134/\nu_0)^2} \]

\[ Z_V = 0.85 + \frac{0.30}{\sqrt{0.8 + (32/\nu)}} \]

\[ Z_R = \left( \frac{3}{R_{Z_{rel}}} \right)^{0.15} \]

where:

\[ \nu_0 = \text{kinematic oil viscosity at } 40^\circ \text{C} \text{ (mm}^2/\text{s}) \]

If the oil temperature before the mesh is considerably higher, e.g., above 80–100°C, it is advised to consider a reduction of \( Z_L \).

For case hardened steels the influence of a high bulk temperature (see 4. Scuffing) should be considered. E.g., bulk temperatures in excess of 120°C for long periods may cause reduced flank surface endurance limits.

\[ R_{Z_{rel}} = \text{the mean roughness between pinion and wheel (after running in) relative to an equivalent radius of curvature at the pitch point } \rho_c = 10 \text{mm} \]

\[ R_{Z_{rel}} = 0.5 \left( R_{Z_1} + R_{Z_2} \right) \left( \frac{10}{\rho_c} \right)^{1/3} \]

\[ R_Z = \text{mean peak to valley roughness (µm) (DIN definition) (roughly } R_Z = 6 R_k) \]

For \( N_1 \leq 10^5 \), \( Z_L, Z_V, Z_R = 1.0 \)

### 2.12 Work Hardening Factor, \( Z_W \)

The work hardening factor, \( Z_W \), accounts for the increase of surface durability of a soft steel gear when meshing the soft steel gear with a surface hardened gear with a smooth surface (\( R_Z \leq 6 \mu m \)).

The following approximation may be used for both the endurance limit and the static strength:

\[ Z_W = 1.2 - \frac{HB - 130}{1700} \]

where

\[ HB = \text{the Brinell hardness of the soft gear.} \]

For \( HB > 470 \), \( Z_W = 1 \).

### 2.13 Size Factor, \( Z_X \)

The size factor accounts for the metallurgical and geometrical size influences on the flank carryability. In particular \( Z_X \) takes subsurface fatigue and yielding into account.

For not surface hardened gears, \( Z_X = 1 \) both for endurance limit and static strength.

For surface hardened gears, \( Z_X \) may be calculated by means of the following method. The main objective is to have a subsurface safety against fatigue (endurance limit) or deformation (static strength) which is at least as high as the safety required for the surface. The considered parameters are the effective radius of curvature at pitch point and the hardness versus depth. In addition, some account is taken for the influence of estimated residual stresses representative for the various hardening processes.

\[ Z_X = \text{applies to the endurance limit} \]

\[ Z_X \times 10^3 = \text{applies to the contact stress } \sigma_{H16} \times 10^3 \text{ cycles} \]

\[ Z_X \times 10^3 = \text{applies to the contact stress } \sigma_{H16} \times 10^3 \text{ cycles} \]

\[ t_{80} = \text{is the min. depth with hardness HV} \geq 550 \]

\[ t_{100} = \text{is the min. depth with hardness HV} \geq 400 \]

\[ t_{150} = \text{is the min. depth with hardness HV} \geq 300 \]

\[ HV_{min} = \text{is the min. depth to HV}_{min} \text{ (applicable to induction and (flame hardening)} \]

\[ HV_{min} = \text{is the min. hardness} \text{ (applicable to induction and (flame hardening)} \]

All depths are after flank grinding. All size factor formulae are for derating purpose when insufficient case depth is used. Thus, when a calculated size factor exceeds 1, the value to be used is 1.

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\[ \text{Fig. 2.1 Examples on hardness profiles} \]

---

\[ \text{DET NORSKE VERITAS} \]
### Hardening processes

<table>
<thead>
<tr>
<th>Hardening process</th>
<th>$Z_X$</th>
<th>$Z_{X10^3}$</th>
<th>$Z_{X10^3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case hardening (lowest value to be used)</td>
<td>$\frac{281550}{\rho_c}$, 0.35 $1550$</td>
<td>$\frac{1550}{\sigma_{\text{Hlim}}}$</td>
<td>$\frac{171550}{\rho_c}$, 0.25 $3100$</td>
</tr>
<tr>
<td></td>
<td>$\frac{174400}{\rho_c}$, 0.4 $1550$</td>
<td>$\frac{2450}{\sigma_{\text{Hlim}}}$</td>
<td>$\frac{854400}{\rho_c}$, 0.4 $3100$</td>
</tr>
<tr>
<td></td>
<td>$\frac{121300}{\rho_c}$, 0.45 $1550$</td>
<td>$\frac{2480}{\sigma_{\text{Hlim}}}$</td>
<td>$\frac{51300}{\rho_c}$, 0.45 $3100$</td>
</tr>
<tr>
<td>Nitriding</td>
<td>$\frac{301400}{\rho_c}$, 0.4 $1280$</td>
<td>$\frac{2600}{\sigma_{\text{Hlim}}}$</td>
<td>$\frac{2600}{\sigma_{\text{Hlim}}}$</td>
</tr>
<tr>
<td>Induction or flame hardening</td>
<td>$\frac{1.1HV_{\min}}{\rho_c}$, 0.4 $1.6 (1.4HV + 350)$</td>
<td>$\frac{HV_{\min}}{\rho_c}$, 0.4 $1.6 (1.4HV + 350)$</td>
<td>$\frac{HV_{\min}}{\rho_c}$, 0.45 $1.6 (1.4HV + 350)$</td>
</tr>
</tbody>
</table>

The values of $Z_X$, $Z_{X10^3}$, and $Z_{X10^3}$ are not to be less than determined by HV of the core material, i.e.

$$Z_X \geq \frac{1.4HV + 350}{\sigma_{\text{Hlim}}}$$

resp.

$$Z_{X10^3} = \frac{1.6 (1.4HV + 350)}{\sigma_{\text{Hlim}}}$$

resp.

$$Z_{X10^3} = \frac{4.5HV}{\sigma_{\text{Hlim}}}$$
3. Calculation of Tooth Strength

3.1 Scope and General Remarks

Part 3 includes the calculation of tooth root strength as limited by tooth root cracking (surface or subsurface initiated) and yielding.

For rim thicknesses $s_R \geq 3.5 \text{ mm}$ the strength is calculated by means of 3.2-3.15. For cylindrical gears the calculation is based on the assumption that the highest tooth root tensile stress arises by application of the force at the outer point of single tooth pair contact (in the transversal plane). For helical gears this position of the applied force on the flank remains, but the tooth root section geometry is calculated for the virtual teeth in the normal section.

The method provides equations for the tooth form factor $Y_F$ and the stress correction factor $Y_S$ with force application at the outer point of single tooth pair contact.

For bevel gears the calculation is based on force application at the tooth tip of the virtual cylindrical gear. Subsequently the stress is converted to load application at the decisive point of contact which is:

- outer point of single pair contact for $\epsilon_F = 0$
- midpoint of zone of contact for $\epsilon_F \geq 1$
- interpolation of above for $0 < \epsilon_F < 1$

In case of a thin annulus or a thin gear rim etc. radial cracking can occur rather than tangential cracking (from root fillet to root fillet). Cracking can also start from the compression fillet rather than the tension fillet. For rim thicknesses $s_R < 3.5 \text{ mm}$ a special calculation procedure is given in 3.16 and 3.17.

A tooth breakage is often the end of the life of a gear transmission. Therefore, a high safety factor $S_F$ against breakage is required.

It should be noted that 3. Calculation of Tooth Strength does not cover fractures caused by:

- oil holes in the tooth root space
- wear steps on the flank
- flank surface distress such as pits or gray staining.

Especially the latter is known to cause oblique fractures starting from the active flank, predominately in spiral bevel gears, but also sometimes in cylindrical gears.

Specific calculation methods for these fractures are not given here, but are under consideration for future revisions. Thus, depending on experience with similar gear designs, limitations other than those outlined in 3. may be applied.

3.2 Basic Equations

The local tooth root stress for pinion resp. wheel is:

For cylindrical gears:

$$\sigma_F = \frac{F_i}{b m_n} Y_F Y_S Y_m K_A K_F \sigma F_a$$

where:

- $F_i$ = Tooth form factor (see 3.3).
- $Y_S$ = Stress correction factor (see 3.4).
- $Y_m$ = Helix angle factor (see 3.5).

For bevel gears:

$$\sigma_F = \frac{F_{rot}}{b m_n} Y_F Y_S Y_m K_A K_F K_{FB} \sigma F_a$$

where:

- $Y_F$ = Tooth form factor, see 3.3.
- $Y_S$ = Stress correction factor, see 3.4.
- $Y_m$ = Contact ratio factor, see 3.5.
- $K_A$ = Bevel correction factor, see 3.7.
- $K_F$ = Load sharing factor, see 3.8.
- $F_{rot}$, $K_A$ etc. see 1.5 to 1.10.
- $b$, see 1.3.

The permissible local tooth root stress for pinion resp. wheel is:

$$\sigma_{FP} = \frac{F_{rot} Y_d Y_N}{S_F} Y_{ RETT} Y\_Rett Y_X Y_C$$

Note that all these factors $Y_d$ etc. are applicable to $3 \times 10^6$ cycles when used in this formula for $\sigma_{FP}$. The influence of lower number of cycles on these factors is covered by the calculation of $Y_N$.

where:

- $Y_d$ = Design factor which accounts for other loads than constant load direction, e.g. idler gears, temporary change of load direction, prestress due to shrinkage, etc. (see 3.10).
- $Y_N$ = Life factor for tooth root stresses related to reference test gear dimensions (see 3.11).
- $S_F$ = Required safety factor acc. to the rules.
- $Y_{ RETT}$ = Relative sensitivity factor of the gear to be determined, related to the reference test gear (see 3.10).
- $Y\_Rett$ = Relative (root fillet) surface condition factor of the gear to be determined, related to the reference test gear (see 3.13).
- $Y_X$ = Size factor (see 3.14).
- $Y_C$ = Case depth factor (see 3.15).

3.3 Tooth Form Factors $Y_F$, $Y_{Fa}$

The tooth form factors $Y_F$ and $Y_{Fa}$ take into account the influence of the tooth form on the nominal bending stress.

$Y_F$ applies to load application at the outer point of single tooth pair contact and is used for cylindrical gears.

$Y_{Fa}$ applies to load application at the tooth tip and is used for bevel gears.

Both $Y_F$ and $Y_{Fa}$ are based on the distance between the contact points of the $30\degree$-tangents at the root fillet of the tooth profile.
Fig. 3.1
Tooth in normal section

Definitions:

\[ Y_F = \frac{h_F}{m_n} \cos \alpha_{Fn} \]

\[ Y_{Fa} = \frac{h_{Fa}}{m_n} \cos \alpha_{Fa} \]

In the case of helical gears, \( Y_F \) and \( Y_{Fa} \) are determined in the normal section, i.e. for a virtual number of teeth. \( Y_{Fa} \) differs from \( Y_F \) by the bending moment arm \( h_{Fa} \). and can be determined by the same procedure as \( Y_F \) with exception of \( h_{Fa} \). For \( h_{Fa} \) the indices \( a \) will change to \( \alpha \) (lip).

The following formulae apply to cylindrical gears, but may also be used for bevel gears when replacing:

- \( m_n \) with \( m_m \)
- \( z_n \) with \( z_m \)
- \( \alpha \) with \( \alpha_{Fn} \)
- \( \beta \) with \( \beta_{m} \)

3.3.1 External gears

3.3.2 Internal gears

The tooth form factor can — as an approximation — be calculated for a substitute gear rack. This substitute gear rack has the form of the basic rack in the normal section, but only the same tooth depth as the internal gear. The angle of force application \( \alpha_{Fn} \) is \( \alpha_{Fn} = \alpha_n \).

\[ G = \rho_{a0} - h_{a0} + x \]

\[ H = \frac{2}{z_n} \left( \frac{\pi}{2} - \frac{E}{m_n} \right) - \frac{\pi}{3} \]

\[ \theta = 2 \frac{G}{z_n} \tan \theta - H \quad \text{(to be solved iteratively)} \]

Tooth root chord \( s_{Fn} \):

\[ \frac{s_{Fn}}{m_n} = z_n \sin \left( \frac{\pi}{3} - \theta \right) + \sqrt{3} \left( \frac{G}{\cos \theta} - \rho_{a0} \right) \]

For bevel gears with a tooth thickness modification:

\( x_{a0} \) affects mainly \( s_{Fn} \), but also \( h_F \) and \( \alpha_{Fn} \). The total influence of \( x_{a0} \) on \( Y_{Fa} \), \( Y_{a0} \) can be approximated by only adding 2 \( x_{a0} \) to \( s_{Fn} \).

Root fillet radius \( \rho_F \) at \( 30^\circ \) tangent:

\[ \frac{\rho_F}{m_n} = \rho_{a0} + \frac{2 G^2}{\cos \theta (z_n \cos \theta - 2 G)} \]

Determination of bending moment arm \( h_F \):

\[ d_n = z_n m_n \]

\[ d_{a0} = d_n + h_a \]

\[ \beta \pi = m_n \cos \alpha_i \cos \beta \]

\[ \theta_{a0} = \cos \alpha_{a0} \]

\[ \theta_{Fa} = \theta_{a0} - \theta_{Fa} \]

\[ \frac{h_{Fa}}{m_n} = \frac{1}{2} \left[ \cos \theta_{Fa} \sin \gamma \tan \alpha_{Fa} \right] \frac{d_{Fa}}{m_n} \]

\[ \frac{h_{Fa}}{m_n} = \Delta h - \left[ \frac{\pi}{4} + \left( h_{a02} - \Delta h \right) \tan \alpha_n \right] \tan \alpha_n - \frac{\rho_{a0}}{2} \]

where:

\[ \rho_{a0} \]
The diameters \( d_{n2} \) and \( d_{c2} \) are to be calculated with the same formulae as for external gears.

### 3.4 Stress Correction Factors \( Y_S \), \( Y_{Sa} \)

The stress correction factors \( Y_S \) and \( Y_{Sa} \) take into account the conversion of the nominal bending stress to the local tooth root stress. Thereby \( Y_S \) and \( Y_{Sa} \) cover the stress increasing effect of the notch (fillet) and the fact that not only bending stresses arise at the root. A part of the local stress is independent of the bending moment arm. This part increases the more the decisive point of load application approaches the critical tooth root section.

Therefore, in addition to its dependence on the notch radius, the stress correction is also dependent on the position of the load application, i.e., the size of the bending moment arm. \( Y_S \) applies to the load application at the outer point of single tooth pair contact, \( Y_{Sa} \) to the load application at tooth tip.

\( Y_S \) can be determined as follows:

\[
Y_S = (1.2 + 0.13 \log_2 \left( \frac{1}{L_{\text{tip}}} \right))
\]

where:

\[
L = \frac{s_{\text{fn}}}{h_F}
\]

\( q_b = \frac{s_{\text{fn}}}{2 \rho F} \) (see 3.3).

\( Y_{Sa} \) can be calculated by replacing \( h_F \) with \( h_{Fa} \) in the above formula.

**Note:**

a) Range of validity \( 1 < q_b < 8 \)

In case of sharper root radii (i.e., produced with tools having too sharp tip radii), \( Y_S \) resp \( Y_{Sa} \) must be specially considered.

b) In case of grinding notches (due to insufficient prehardenability of the hob), \( Y_S \) resp \( Y_{Sa} \) can rise considerably, and must be multiplied with:

\[
1.3 \left( 1.2 - 0.6 \sqrt{ \frac{1}{\rho_b} } \right)
\]

where:

\( \rho_b \) = radius of the grinding notch

3.5 Contact Ratio Factor \( Y_c \)

The contact ratio factor \( Y_c \) covers the conversion from load application at the tooth tip to the load application at the decisive point for bevel gears.

### 3.6 Helix Angle Factor \( Y_\beta \)

The helix angle factor, \( Y_\beta \), takes into account the difference between the helical gear and the virtual spur gear in the normal section on which the calculation is based in the first step. In this way it is accounted for that the conditions for tooth root stresses are more favourable because the lines of contact are sloping over the flank.

The following may be used:

\[
Y_\beta = 1 - \frac{\beta}{100} \quad \text{valid for } \beta \geq 1
\]

\[
Y_\beta = 1 - \frac{\beta}{100} \quad \text{valid for } \beta < 1
\]

### 3.7 Bevel Gear Factor \( Y_K \)

The bevel gear factor accounts for differences between bevel and cylindrical gears (smaller values of \( h' \), inclined lines of contact).

\[
Y_K = \frac{1}{2} + \frac{b}{4 h} + \frac{h'}{4 h}
\]

where:

\( h' = h \cos \beta_{hm} \)

\( h \) and \( \beta_{hm} \), see 1.4

### 3.8 Load Sharing Factor \( Y_{LS} \)

The load sharing factor accounts for load sharing between 2 or more pairs of teeth for \( \sigma_g > 2 \).

\[
Y_{LS} = Z_{LS}
\]

3.9 Values of Endurance Limit, \( \sigma_{FE} \)

\( \sigma_{FE} \) is the local tooth root stress which the material can endure permanently with 99% survival probability. \( 3 \times 10^9 \) load cycles is regarded as the beginning of the endurance limit or the lower knee of the \( \sigma-N \) curve. \( \sigma_{FE} \) is defined as the unidirectional pulsating stress with a minimum stress of zero (disregarding residual stresses due to heat treatment). Other stress conditions such as alternating or pre-stressed etc. are covered by the conversion factor \( Y_\sigma \).

\( \sigma_{FE} \) can be found by pulsating tests or gear running tests for any material, in any condition. If the approval of the gear is to be based upon the results of such tests, all details on the testing conditions have to be approved by the Society. Further, the tests may have to be made under the Society’s supervision.
3.10 Design Factor, Y_d

The design factor, Y_d, takes into account the influence of other working stress conditions than pure pulsations (R = 0), such as e.g. load reversings, idler gears, planets and shrinkfitted gears.

Y_d is defined as the ratio between the endurance (or static) strength with a stress ratio R ≠ 0, and the endurance (or static) strength with R = 0.

Y_d applies only to positive (tensile) stresses and is therefore suitable for comparison between the calculated (positive) working stress \( \sigma \) and the permissible stress \( \sigma_{pp} \) calculated with Y_d.

The following method may be used within a stress ratio -1.2 < R < 0.5:

### 3.10.1 For idlers and planets

\[
Y_d = \frac{1}{1 - \frac{R}{1 + M}}
\]

where

- \( R = \) stress ratio = min. stress divided by max. stress.
- For designs with the same force applied on both forward- and back-flank, R may be assumed to -1,2.
- For designs with considerably different forces on forward- and back-flank, such as e.g. a marine propulsion wheel with a power take off pinion, R may be assessed as:

\[
force \ per \ unit \ facet \ width \ of \ the \ p.t.o. = -1.2 \ force \ per \ unit \ facet \ width \ of \ the \ main \ branch
\]

M considers the mean stress influence on the endurance (or static) strength amplitudes.

M is defined as the reduction of the endurance strength amplitude for a certain increase of the mean stress divided by that increase of the mean stress.

Following M values may be used:

<table>
<thead>
<tr>
<th>Material</th>
<th>Endurance limit</th>
<th>Static strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case hardened</td>
<td>0.8 - 0.15 ( Y_a )</td>
<td>0.7</td>
</tr>
<tr>
<td>Shot peened</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Nitrided</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Induction or flame hardenened</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Not surface hardened steel</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>Cast steel</td>
<td>0.4</td>
<td>0.6</td>
</tr>
</tbody>
</table>

The listed M values for the endurance limit are independent of the fillet shape \( Y_a \), except for case hardening. In principle there is a dependency of the static notch sensitivity factor (and thereby indirectly \( Y_a \)), but wide variations usually only occur for case hardening, e.g. smooth semicircular fillets versus grinding notches.

### 3.10.2 For gears with periodical change of rotational direction

For case hardened gears with full load applied periodically in both directions, such as side thrusters, the same formula for \( Y_d \) as for idlers (with \( R = -1,2 \)) may be used together with the M values for endurance limit. This simplified approach is valid when the number of changes of direction exceeds 100 and the total number of load cycles exceeds 3·10^6.

For gears of other materials, \( Y_d \) will normally be higher than for a pure idler, provided the number of changes of direction is below 3·10^6. A linear interpolation in a diagram with logarithmic number of changes of direction may be used, i.e. from \( Y_d = 0,9 \) with one change to \( Y_d \) (idler) for 3·10^6 changes. This is applicable to \( Y_d \) for endurance limit.

For static strength, use \( Y_d \) as for idlers.

For gears with occasional full load in reversed direction, such as the main wheel in a reversing gear box, \( Y_d = 0,9 \) may be used.
3.10.3 For gears with shrinkage stresses and unidirectional load

For endurance strength:

\[ Y_d = 1 - \frac{2 M}{1 + M} \frac{1 - R}{1 - R \frac{M}{1 + M}} \]

\[ \sigma_{FE} \] is the endurance limit for \( R = 0 \).

For static strength, subtract \( \sigma_{FE} \) from the static strength.

\( \sigma_{st} \) is the shrinkage stress in the fillet (30° tangent) and may be found by multiplying the nominal tangential (hoop) stress with a stress concentration factor

\[ 1.5 - \frac{2 \rho_F}{m_n} \]

3.10.4 For shrunkfitted idlers and planets

When combined conditions apply, such as idlers with shrinkage stresses, the design factor for endurance strength is:

\[ Y_d = 1 - \frac{2 M}{1 + M} \frac{1 - R}{1 - R \frac{M}{1 + M}} \]

Symbols as above, but note that the stress ratio \( R \) in this particular connection should disregard the influence of \( \sigma_{st} \), i.e. \( R \) normally equal \(-1,2\).

For static strength:

\[ Y_d = 1 - \frac{1}{1 - R \frac{M}{1 + M}} \]

and subtraction of \( \sigma_{st} \) from the static strength calculated with this \( Y_d \) (before division with the safety factor), see 3.11b.

3.10.5 Additional requirements for peak loads

The total stress range \( (\sigma_{max} - \sigma_{min}) \) in a tooth root fillet is not to exceed:

\[ \frac{2.25 \sigma_y}{SF} \quad \text{for not surface hardened fillets} \]

\[ \frac{5 \rm{HV}}{SF} \quad \text{for surface hardened fillets} \]

3.11 Life Factor, \( Y_N \)

The life factor, \( Y_N \), takes into account that, in the case of limited life (number of cycles), a higher tooth root stress can be permitted and that lower stresses may apply for very high number of cycles.

Decisive for the strength at limited life is the \( \sigma - N \) - curve of the respective material for given hardening, module, fillet radius, roughness in the tooth root, etc. i.e. the factors \( Y_{relT}, Y_{relT}, Y_x \) and \( Y_d \) have an influence on \( Y_N \).

If no \( \sigma - N \) - curve for the actual material and hardening etc. is available, the following method may be used.

Determination of the \( \sigma - N \) - curve:

\[ a) \] Calculate the permissible stress \( \sigma_{FP} \) for the beginning of the endurance limit (\( 3 \times 10^6 \) cycles), including the influence of all relevant factors as \( SF, Y_{relT}, Y_{relT}, Y_x, Y_d \) and \( Y_c \).

\[ b) \] Calculate the permissible «static» stress (\( \leq 10^3 \) load cycles) including the influence of all relevant factors as \( SF, Y_{relT}, Y_d \) and \( Y_c \):

\[ \sigma_{PP} = \frac{1}{SF} (\sigma_{Fat} Y_d Y_{relT} Y_c - \sigma_{fit}) \]

where \( \sigma_{fit} \) is the local tooth root stress which the material can resist without cracking (surface hardened materials) or unacceptable deformation (not surface hardened materials) with 99% probability.

<table>
<thead>
<tr>
<th>Alloyed case hardened steel</th>
<th>2.5 ( \sigma_{FE} ) but not more than 2300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitriding, steel, gas nitrided (surface hardness 650—800 HV)</td>
<td>1250</td>
</tr>
<tr>
<td>Alloyed quenched and tempered steel, both or gas nitrided (surface hardness 500—700 HV)</td>
<td>1050</td>
</tr>
<tr>
<td>Alloyed quenched and tempered steel, flame or induction hardened (fillet surface hardness 500—650 HV)</td>
<td>1.8 ( HV + 800 )</td>
</tr>
<tr>
<td>Forged or rolled steel with not surface hardened fillets, the smaller value of</td>
<td>1.8 ( \sigma_B ) or 2.25 ( \sigma_y )</td>
</tr>
<tr>
<td>Cast steel with not surface hardened fillets, the smaller value of</td>
<td>1.2 ( \sigma_B ) or 2.25 ( \sigma_y )</td>
</tr>
</tbody>
</table>

1) This is valid for a fillet surface hardness of 58—63 HRC. In case of lower fillet surface hardness than 58 HRC, \( \sigma_{fit} \) is to be reduced with 30 (58—HRC) where HRC is the actual hardness. Shot peening or grinding notches are not considered to have any significant influence on \( \sigma_{fit} \).

\[ c) \] Calculate \( Y_N \) as:

\[ N_l > 3 \times 10^6 \]

\[ Y_N = \frac{1}{N_l} \]

\[ Y_N = \left( \frac{3 \times 10^6}{N_l} \right) ^{0.02} \]

The \( Y_N = \frac{1}{N_l} \) from \( 3 \times 10^6 \) may only be used when optimum material cleanliness applies.

\[ 10^3 < N_L < 3 \times 10^6 \]

\[ Y_N = \left( \frac{3 \times 10^6}{N_L} \right) ^{\text{exp}} \]

\[ \text{exp} = 0.2876 \log \frac{\sigma_{FP} \text{ for } 10^3 \text{ cycles}}{\sigma_{FP} \text{ for } 3 \times 10^6 \text{ cycles}} \]

\[ N_L < 10^3 \]

\[ Y_N = \frac{\sigma_{FP} \text{ for } 10^3 \text{ cycles}}{\sigma_{FP} \text{ for } 3 \times 10^6 \text{ cycles}} \]

or simply \( \sigma_{FP} \) as mentioned in b).
3.12 Relative Notch Sensitivity Factor, $Y_{RSET}$

The dynamic (resp. static) relative notch sensitivity factor, $Y_{RSET}$, indicates to which extent the theoretically concentrated stress lies above the endurance limits (resp. static strengths) in the case of fatigue (resp. overload) breakage.

$Y_{RSET}$ is a function of the material and the relative stress gradient. It differs for static strength and endurance limit.

The following method may be used:

For endurance limit:

- For not surface hardened fillets:
  $$Y_{FRSET} = 1 + 0.036 (q_s - 2.5) \left(1 - \frac{\sigma_y}{1200}\right)$$
- For all surface hardened fillets except nitrided:
  $$Y_{FRSET} = 0.956 + 0.0234 \sqrt{1 + q_s}$$
- For nitrided fillets:
  $$Y_{FRSET} = 0.79 + 0.112 \sqrt{1 + q_s}$$

For static strength:

- For not surface hardened fillets (forged or rolled steels):
  $$Y_{FRSET} = 0.86 + 0.07 Y_S$$
- For not surface hardened fillets (cast steels):
  $$Y_{FRSET} = 0.4 + 0.3 Y_S$$
- For surface hardened fillets except nitrided:
  $$Y_{FRSET} = 0.6 + 0.2 Y_S$$
- For nitrided fillets:
  $$Y_{FRSET} = 0.6 + 0.2 Y_S$$

3.13 Relative Surface Condition Factor, $Y_{RSF}$

The relative surface condition factor, $Y_{RSF}$, takes into account the dependence of the tooth root strength on the surface condition in the tooth root fillet, mainly the dependence on the peak to valley surface roughness.

$Y_{RSF}$ differs for endurance limit and static strength.

The following method may be used:

For endurance limit:

- For surface hardened steels and alloyed quenched and tempered steels except nitrided:
  $$Y_{RSF} = 1.675 - 0.53 (R_y + 1)^{0.01}$$
- For carbon steels:
  $$Y_{RSF} = 5.3 - 4.2 (R_y + 1)^{0.01}$$
- For nitrided steels:
  $$Y_{RSF} = 4.3 - 3.26 (R_y + 1) 0.005$$

For static strength:

$$Y_{RSF} = 1$$

A fillet without any longitudinal machining traces, $R_y = R_s$.

3.14 Size Factor, $Y_X$

The size factor, $Y_X$, takes into account the decrease of the strength with increasing size. $Y_X$ differs for endurance limit and static strength.

The following may be used:

<table>
<thead>
<tr>
<th>Hardening process</th>
<th>$t$</th>
<th>for endurance limit</th>
<th>for static strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case hardening</td>
<td>150</td>
<td>1900</td>
<td>$\frac{1900}{Y_{1.0}}$</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>1200</td>
<td>$\frac{1200}{Y_{1.0}}$</td>
</tr>
<tr>
<td>Nitriding</td>
<td>150</td>
<td>800</td>
<td>$\frac{800}{Y_{1.0}}$</td>
</tr>
<tr>
<td>Induction- or flame hardening</td>
<td>$HV_{min}$</td>
<td>$\frac{2.3 HV_{min}}{Y_{1.0}}$</td>
<td></td>
</tr>
</tbody>
</table>

For endurance limit:

<table>
<thead>
<tr>
<th>$X$</th>
<th>for $m_n \leq 5$</th>
<th>generally</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_X = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y_X = 1.03 - 0.006 m_n$</td>
<td>for $5 &lt; m_n &lt; 30$</td>
<td></td>
</tr>
<tr>
<td>$Y_X = 0.85$</td>
<td>for $m_n \geq 30$</td>
<td></td>
</tr>
<tr>
<td>$Y_X = 1.05 - 0.01 m_n$</td>
<td>for $5 &lt; m_n &lt; 25$</td>
<td></td>
</tr>
<tr>
<td>$Y_X = 0.8$</td>
<td>for $m_n \geq 25$</td>
<td></td>
</tr>
</tbody>
</table>

For static strength:

$$Y_X = 1$$

for all $m_n$ and all materials.

3.15 Case Depth Factor, $Y_C$

The case depth factor, $Y_C$, takes into account the influence of hardening depth on tooth root strength.

$Y_C$ applies only to surface hardened tooth roots, and is different for endurance limit and static strength.

In case of insufficient hardening depth, fatigue cracks can develop in the transition zone between the hardened layer and the core. For static strength, yielding shall not occur in the transition zone as this would alter the surface residual stresses and therewith also the fatigue strength.

The major parameters are case depth, stress gradient, permissible surface resp. subsurface stresses, and subsurface residual stresses.

The following simplified method for $Y_C$ may be used.

$Y_C$ consists of a ratio between permissible subsurface stress (incl. influence of expected residual stresses) and permissible surface stress. This ratio is multiplied with a bracket containing the influence of case depth and stress gradient. (The empirical numbers in the bracket are based on a high number of teeth, and are somewhat on the safe side for low number of teeth.)

$Y_C$ is the minimum of 1.0 and the minimum value from the following:

$$Y_C \approx \text{const.} \frac{r_{FE}}{\rho_{FF} + 0.2 m_n} \left(1 + \frac{3 t}{\rho_{FF} + 0.2 m_n}\right)$$

where const. and $t$ are connected as:

<table>
<thead>
<tr>
<th>Hardening process</th>
<th>$t$</th>
<th>for endurance limit</th>
<th>for static strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case hardening</td>
<td>150</td>
<td>1900</td>
<td>$\frac{1900}{Y_{1.0}}$</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>1200</td>
<td>$\frac{1200}{Y_{1.0}}$</td>
</tr>
<tr>
<td>Nitriding</td>
<td>150</td>
<td>800</td>
<td>$\frac{800}{Y_{1.0}}$</td>
</tr>
<tr>
<td>Induction- or flame hardening</td>
<td>$HV_{min}$</td>
<td>$\frac{2.3 HV_{min}}{Y_{1.0}}$</td>
<td></td>
</tr>
</tbody>
</table>
3.16 Stress in Thin Rims

For rim thicknesses \( r_s < 3.5 \, m_n \) the safety against rim cracking has to be checked.

The following simplified approach may be used.

3.16.1 General

The stresses in the standardized 30° tangent section, tension side, are slightly reduced due to increasing stress correction factor with decreasing relative rim thickness \( s_R/m_n \). On the other hand, during the complete stress cycle of the fillet, a certain amount of compressive stresses are also introduced.

The complete stress range remains approximately constant. Therefore, the standardized calculation of stresses at the 30° tangent may be retained for thin rims as one of the necessary criteria.

The maximum stress range for thin rims usually occurs at the 60°—80° tangents, both for «tension» and «compression» side. The following method assumes the 75° tangent to be the decisive. Therefore, in addition to the a.m. criterion applied at the 30° tangent, it is necessary to evaluate the max and min stresses at the 75° tangent for both «tension» (loaded flank) fillet and «compression» (back-flank) fillet. For this purpose the whole stress cycle of each fillet should be considered, but usually the following simplification is justified:

\[
Y_C = 1 - \left( \frac{1.550 \, \text{max} \, m_n}{m_n} - 0.25 \right)
\]

3.16.2 Stress concentration factors at the 75° tangents

The nominal rim stress consists of bending stresses due to local bending moments, tangential stresses due to the tangential force \( F_t \), and radial shear stresses due to \( F_r \).

The major influence is given by the bending stresses. The influence of the tangential stresses is minor, and even though its stress concentration factor is slightly higher than for bending, it is considered to be safe enough when the sum of these nominal stresses are combined with the stress concentration factor for bending. The influence of the radial shear stress is neglected.

The stress concentration factor relating nominal rim stresses to local fillet stresses at the 75° tangent may be calculated as:

\[
Y_{75} = \frac{3}{1 + \left( \frac{s_R}{m_n} \right)^{1.5}}
\]

3.16.3 Nominal rim stresses

The bending moment applied to the rim consists of a part of the tooth tilt moment \( F_t \) (15° + 0.5 sq) and the bending caused by the radial force \( F_r \).

The sectional moduli which are used for determination of the nominal bending stresses are not necessarily the same for the 2 a.m. bending moments. If flanges, webs, etc. outside the toothed section contribute to stiffening the rim against various deflections, the influence of these stiffeners should be considered. E.g. an end flange will have an almost negligible influence on the effective sectional modulus for the stresses due to tooth tilt as the deflection caused by the tilt moment is rather small and would not much involve the flange. On the other hand, the radial forces, as for instance from the meshes in an annulus, would cause considerable radial deflections which the flange might restrict to a substantially amount. When considering the stiffening of such flanges or webs on basis of simplified models, it is advised to use an effective rim thickness \( s_R = s_R + 0.2 \, m_n \) for the area moment of inertia of the rim (toothed part) cross section.

For a high number of rim teeth, it may be assumed that the rim bending moments in the fillets adjacent to the loaded tooth are of the same magnitude as right under the applied force. This assumption is reasonable for an annulus, but rather much on the safe side for a hollow pinion.

The influence of \( F_t \) on the nominal tangential stress is simplified by half of it for compressive stresses \( (\sigma_1) \) and the other half for tensile stresses \( (\sigma_2) \). Applying these assump-
tions, the nominal rim stresses adjacent to the loaded tooth are:

\[
\sigma_1 = \frac{-0.5 F_1 (b_R + 0.5 s_R)}{W_T} F_R f(\beta) - \frac{F_1}{2 A}
\]

\[
\sigma_2 = \frac{-0.5 F_1 (b_R + 0.5 s_R)}{W_T} F_R f(\beta) + \frac{F_1}{2 A}
\]

where \( \sigma_1, \sigma_2 \) see Fig. 3.3.

\[A = \text{minimum area of cross section (usually } b_R s_R).\]

\[W_T = \text{the sectional modulus of rim with respect to tooth tilt moment (usually } b_R s_R/6).\]

\[W_R = \text{the sectional modulus of rim including the influence of stiffeners as flanges, webs etc. } (W_R \geq W_T).\]

\[b_R = \text{the width of the rim.}\]

\[R = \text{the radius of the neutral axis in the rim.}\]

\[f(\beta) = \text{a function for bending moment distribution around the rim.}\]

For a rim (pinion) with one mesh only, the \( f(\beta) \) at the position of load application is 0.24.

For an annulus with 3 or more meshes, \( f(\beta) \) at the position of each load application is approx.:

- 3 planets \( f(\beta) = 0.19 \)
- 4 planets \( f(\beta) = 0.14 \)
- 5 planets \( f(\beta) = 0.11 \)
- 6 planets \( f(\beta) = 0.09 \)

It must be checked if the maximum (tensile) stress in the compression fillet really occurs when the fillet is adjacent to the loaded tooth. In principle, the stress variation through a complete rotation should be considered, and the max value used. The max value is usually never less than 0. For an annulus, e.g. the tilt moment is zero in the mid position between the planet meshes, whilst the bending moment due to the radial forces is half of that at the mesh but with opposite sign.

If these formulae are applied to idler gears, as e.g. planets, the influence of nominal tangential stresses must be corrected by deleting \( F_1/2 A \) for \( \sigma_1 \), and using \( F_1/2 A \) for \( \sigma_2 \). Further, the influence of \( F_R \) on the nominal bending stresses is usually negligible due to the planet bearing support.

3.16.4 Root Fillet Stresses

Determination of min and max stresses in the «tension» fillet:

Minimum stress:

\[\sigma_{FTmin} = K \sigma_1 Y_{75}\]

Maximum stress:

\[\sigma_{FTmax} = K \sigma_2 Y_{75} + \sigma_F Y_{cor} 0.36\]

where \( \sigma_3 \) is an empirical factor relating the tension stresses \( \sigma_F \) at the 30° tangent to the part of the compression stresses at the 75° tangent which add to the rim related stresses. \( \sigma_3 \) also takes into account that full superposition of nominal stresses times stress concentration factors from both «sides of the corner fillets» would result in too high stresses.

\[K = K_A K_p K_v K_{sR} K_{R} \]

Determination of min and max stresses in the «compression» fillet:

Minimum stress:

\[\sigma_{FCmin} = K \sigma_1 Y_{75} + \sigma_F Y_{cor} 0.36\]

Maximum stress:

\[\sigma_{FCmax} = K \sigma_2 Y_{75}\]

where 0.36 is an empirical factor relating the tension stresses \( \sigma_F \) at the 30° tangent to the part of the compression stresses at the 75° tangent which add to the rim related stresses.

For gears with reversed loads as idler gears and planets there is no distinct «tension» or «compression» fillet. The minimum stress \( \sigma_{Fmin} \) is the minimum of \( \sigma_{FTmin} \) and \( \sigma_{FCmin} \) (usually the latter is decisive). The maximum stress \( \sigma_{Fmax} \) is the maximum of \( \sigma_{FTmax} \) and \( \sigma_{FCmax} \) (usually the former is decisive).

3.17 Permissible Stresses in Thin Rims

3.17.1 General

The safety against fatigue fracture resp. overload fracture is to be at least at the same level as for solid gears. The «ordinary» criteria at the 30° tangent apply as given in 3.1 through 3.1.5.

Additionally the following criteria at the 75° tangent may apply.

3.17.2 For \( > 3 \times 10^6 \) cycles

The permissible stresses for the «tension» fillets and for the «compression» fillets are determined by means of a relevant fatigue diagram.

If the actual tooth root stress (tensile or compressive) exceeds the yield strength of the material, the induced residual stresses are to be taken into account.

For determination of permissible stresses the following is defined:

\[R = \text{stress ratio, i.e. } \frac{\sigma_{FTmin}}{\sigma_{FTmax}} \text{ resp., i.e. } \frac{\sigma_{FCmin}}{\sigma_{FCmax}}\]

\[\Delta \sigma = \text{stress range, i.e. } \frac{\sigma_{FTmax} - \sigma_{FTmin}}{\sigma_{FCmax} - \sigma_{FCmin}}\]

(For idler gears and planets \( R = \frac{\sigma_{Fmin}}{\sigma_{Fmax}} \))

The permissible stress range \( \Delta \sigma_p \) for the «tension» resp. «compression» fillets can be calculated as:

\[\Delta \sigma_p = \frac{1.3}{l + 0.3} \frac{1 + R}{1 - R} \sigma_{FP}\]

For \( -\infty < R < -1 \)

\[\Delta \sigma_p = \frac{1.3}{l + 0.15} \frac{1 + R}{1 - R} \sigma_{FP}\]

where:

\[\sigma_{FP} \text{ see 3.2, determined for unidirectional stresses (} Y_d = 1).\]

If the yield strength \( \sigma_y \) is exceeded in either tension or compression, residual stresses are induced. This may be
considered by correcting the stress ratio R for the respective fillets (tension or compression).

E.g. if $|\sigma_{Fc_{min}}| > \sigma_y$
(i.e. exceeded in compression), the difference
$\Delta = |\sigma_{Fc_{min}}| - \sigma_y$
affects the stress ratio as

$$R = \frac{\sigma_{Fc_{min}} + \Delta}{\sigma_{Fc_{max}} + \Delta} = \frac{-\sigma_y}{\sigma_{Fc_{max}} + \Delta}$$

Similarly the stress ratio in the tension fillet may require correction.

If the yield strength is exceeded in tension, $\sigma_{Ft_{max}} > \sigma_y$, the difference $\Delta = \sigma_{Ft_{max}} - \sigma_y$ affects the stress ratio as

$$R = \frac{\sigma_{Ft_{min}} - \Delta}{\sigma_{Ft_{max}} - \Delta} = \frac{\sigma_{Ft_{min}} - \Delta}{\sigma_y}$$

Checking for possible exceeding of the yield strength has to be made with the highest torque, and with the $K_{Ap}$ if this exceeds $K_A$.

3.17.3 For $\leq 10^3$ cycles
The permissible stress range $\Delta\sigma_p$ is not to exceed:
For $R > -1$ $\Delta\sigma_p = \frac{1,5}{1 + 0,5 + \frac{1 + R}{1 - R}} \sigma_{fp}$

For $-\infty < R < -1$ $\Delta\sigma_p = \frac{1,5}{1 + 0,25 + \frac{1 + R}{1 - R}} \sigma_{fp}$

For all values of $R$, $\Delta\sigma_p$ is limited by:

- not surface hardened: $\frac{2,25 \sigma_y}{S_F}$
- surface hardened: $\frac{S_H V}{S_F} Y_C$

Definition of $\Delta\sigma$ and R, see 3.17.2, with particular attention to possible correction of R if the yield strength is exceeded. $\sigma_{fp}$ see 3.2, determined for unidirectional stresses ($Y_d = 1$) and $< 10^3$ cycles.

3.17.4 For $10^3 < \text{cycles} < 3 \cdot 10^6$
$\Delta\sigma_p$ is to be determined by linear interpolation a log-log diagram.
$\Delta\sigma_p$ at $N_L$ load cycles is:

$$\Delta\sigma_p_{N_L} = \left( \frac{3 \cdot 10^6}{N_L} \right)^{exp} \Delta\sigma_p \cdot 3 \cdot 10^6$$

$exp = 0,2876 \log \frac{\Delta\sigma_p \cdot 10^3}{\Delta\sigma_p \cdot 3 \cdot 10^6}$
4. Calculation of Scuffing Load Capacity

4.1 Introduction

High surface temperatures due to high loads and sliding velocities can cause lubricant films to break down. This seizure or welding together of areas of tooth surfaces is termed scuffing.

In contrast to pitting and fatigue breakage which show a distinct incubation period, a single short overloading can lead to a scuffing failure. In the ISO draft technical report (previously ISO-DIS 6336 part 4) two criteria are mentioned. The methods used in this Classification Note are based on the principles of these a.m. criteria, but are essentially modified. The criteria are:

a) The flash temperature criterion; based on contact temperatures which vary along the path of contact.

b) The integral temperature criterion; based on the weighted average of the contact temperatures along the path of contact.

Both criteria are to be fulfilled.

The flash temperature criterion is normally the decisive, and the integral temperature criterion is included mainly for comparison purpose. However, for gears with high oil temperatures the flash temperature criterion can be fulfilled even though the difference between the scuffing temperature and the flash temperature is relatively small. For such gears the integral temperature criterion should be preferred.

In the Rules, required safety factors are given as:

- Propulsion gears $S_S \geq 1.5$
- Auxiliary gears $S_S \geq 1.4$

It should be noted that these safety factors are expressed as the ratio between 2 temperatures. In special cases this ratio (safety factor) may be fulfilled, but the absolute difference between scuffing temperature and actual contact temperature may still be small. In addition to the safety factor it is therefore required a minimum temperature difference of 50°C for the flash temperature criterion.

Note: Bulk temperatures in excess of 120°C for long periods may have an adverse effect on the surface durability, see 2.11.

4.2 General criteria

In no point along the path of contact the local contact temperature may exceed the permissible temperature, i.e.:

$$\theta_B \leq \frac{\theta_S - \theta_{\text{oil}}}{S_S} + \theta_{\text{oil}}$$

$$\theta_B \leq \theta_S - 50$$

$$\theta_{\text{int}} \leq \frac{\theta_S}{S_S}$$

where:

$$\theta_B = \text{max. contact temperature along the path of contact.}$$

$$\theta_{\text{MB}} = \text{bulk temperature, see 4.3.4.}$$

$$\theta_{\text{flmax}} = \text{max. flash temperature along the path of contact, see 4.4.}$$

$$\theta_S = \text{scuffing temperature, see below.}$$

$$\theta_{\text{oil}} = \text{oil temperature before it reaches the mesh (max. applicable for the actual load case to be used, i.e. normally alarm temperature, except for ice classes where a max. expected temperature applies).}$$

$$S_S = \text{required safety factor acc. to the Rules.}$$

$$\theta_{\text{int}} = \text{integral temperature.}$$

$$\theta_{\text{int}} = \theta_{\text{MC}} + 1.5 \theta_{\text{flint}}$$

$$\theta_{\text{MC}} = \text{bulk temperature, see 4.3.4.}$$

$$\theta_{\text{flint}} = \text{mean flash temperature along the path of contact, see 4.5.}$$

The scuffing temperature $\theta_S$ may be calculated as:

$$\theta_S = 80 + (0.85 + 1.1 X_{wrelT}) FZG^2 X_L$$

where:

$$X_{wrelT} = \text{relative welding factor.}$$

$$FZG = \text{load stage acc. to FZG-Test A/8.3/90.}$$

(Note: This is the load stage where scuffing occurs. However, it is assumed that the oil somewhat deteriorates during an oil shift interval. Therefore calculations are to be made with one load stage less than the specification.) $X_L = \text{lubricant factor.}$

- 1.0 for mineral oils.
- 0.8 for polyalkylene glycols.
- 0.7 for non-water-soluble polyglycols.
- 0.6 for water-soluble polyglycols.
- 1.5 for traction fluids.
- 1.3 for phosphate esters.

Application of other test methods such as the Ryder, the FZG-Ryder R/46,5/74, and the FZG L-42 Test 141/19,5/110 may be specially considered.

For high speed gears with very short time of contact $\theta_S$ may be specially considered.
4.3 Common Influence Factors

4.3.1 Coefficient of friction
For calculation of both $\vartheta_B$ and $\vartheta_{int}$ the following coefficient of friction may apply:

$$\mu = 0.06 \left( \frac{w_{BL} \cdot \rho \cdot \rho_{redC}}{\nu \cdot \Sigma C} \right)^{0.2} \eta_{oil}^{-0.05} R_o^{0.25} X_1,$$

where:

- $w_{BL} = \text{specific tooth load (N/mm)}$
- $\nu \Sigma C = \text{sum of tangential velocities at pitch point}$
- $\rho \cdot \rho_{redC} = \text{relative radius of curvature (transversal plane) at the pitch point}$

**Cylindrical gears**

$$w_{BL} = \frac{F_{BL}}{b} \cdot K_A \cdot K_y \cdot K_{H1} \cdot K_{Ha} \quad \text{(see 1)}$$

$$\nu \Sigma C = 2 \cdot v \cdot \sin \alpha_{wl}$$

$$\rho \cdot \rho_{redC} = \rho \cdot \cos \beta_b$$

**Bevel gears**

$$w_{BL} = \frac{F_{BL}}{0.85 \cdot b} \cdot K_A \cdot K_y \cdot K_{H1} \cdot K_{H2} \cdot K_{Ha} \quad \text{(see 1)}$$

$$\nu \Sigma C = 2 \cdot \nu_{int} \cdot \sin \alpha_{vl}$$

$$\rho \cdot \rho_{redC} = \rho \cdot \cos \beta_{vm}$$

$\eta_{oil} = \text{dynamic viscosity (mPa s) at } \vartheta_{oil} \text{ calculated as}$

$$\eta_{oil} = \frac{\mu_{oil} \cdot \rho}{1000}$$

where $\rho$ in kg/m$^3$ approximated as

$$\rho = \rho_{1.5} = (9 - 15) \cdot 0.0007$$

and $\nu_{oil}$ is kinetic viscosity at $\vartheta_{oil}$ and may be calculated by means of the following equation:

$$[\log \log (\nu_{oil} + 0.8) - \log \log (\nu_{100} + 0.8) + \frac{\log 373 - \log (273 + \vartheta_{oil})}{\log 373 - \log 313} \cdot (\log \log (\nu_{0.8} + 0.8) - \log \log (\nu_{100} + 0.8))]$$

$R_o = \text{composite arithmetic mean roughness (micron) of pinion and wheel calculated as}$

$$R_o = 0.5 \cdot (R_{al} + R_{aw})$$

This is defined as the roughness on the new flanks i.e. as manufactured.

$X_1 = \text{see 4.2.}$

4.3.2 Effective tip relief $C_{elr}$

$C_{elr}$ is the effective tip relief, that amount of tip relief which compensates for the elastic deformation of the gear mesh, i.e. zero load at the tooth tip. It is assumed (simplified) to be the equal for both pinion and wheel.

### Cylindrical gears

For helical:

$$C_{elr} = \frac{F_{BL} \cdot K_A \cdot K_y}{b \cdot c_r} \quad \text{(see 1)}$$

For spur:

$$C_{elr} = \frac{F_{BL} \cdot K_A \cdot K_y}{b \cdot c_r} \quad \text{(see 1)}$$

### Bevel gears

$$C_{elr} = \frac{F_{BL} \cdot K_A}{14.7 \cdot b_{eff}} \quad \text{(see 1)}$$

where:

$\beta_{eff} = \text{active facewidth (mm). If } \beta_{eff} > 0.85 \cdot b, \text{ use } \beta_{eff} = 0.85 \cdot b. \text{ See also comments on accumulation of contact patterns in 1.9.}$

4.3.3 Tip relief and extension

**Cylindrical gears**

The extension of the tip relief is not to result in an effective contact ratio $\vartheta_{tr} < 1$ when the gear is unloaded (exceptions to this may only apply for applications where the gear is not to run at light loads). This means that the unrelieved part of the path of contact is to be minimum $p_m$. It is further assumed that this unrelieved part is placed centrally on the path of contact.

If root relief applies, it has to be calculated as equivalent tip relief, see «Bevel gears» below, but replace $u_{40}$ with $u_e$ with $u_v$ with $u$, $m_{min}$ with $m_0$, and $\alpha_{vl}$ with $\alpha_{wl}$.

**Bevel gears**

Normally bevel gears have heightwise crowning, i.e. no distinct relieved/unrelieved area. This may be treated as tip and root relief. For calculation purposes the root relief is combined with the tip relief of the mating member into an equivalent tip relief. Following approximations may be used:

$$C_{e1eq} = C_{elr} + C_{root2} \left( A_2 - 1 \right)^2$$

$$C_{e2eq} = C_{elr} + C_{root1} \left( A_1 - 1 \right)^2$$

where

$$A_2 = \frac{u_{40} - 2 \cdot u_e - \sqrt{u_e^2 \cdot \cos^2 \alpha_{vl} + \sin^2 \alpha_{vl} \cdot (u_e - \Gamma_e)^2}}{2 \cdot m_{min}}$$

$$A_1 = \frac{u_{40} - 2 \cdot u_e - \sqrt{u_e^2 \cdot \cos^2 \alpha_{vl} + \sin^2 \alpha_{vl} \cdot (1 + \Gamma_A)^2}}{2 \cdot m_{min}}$$

Throughout the following $C_{e1}$ and $C_{e2}$ mean the equivalent tip relieves $C_{e1eq}$ and $C_{e2eq}$.
If no design tip relief is specified, use the running amount, see 1.12.

4.3.4 Bulk temperature
The bulk temperature may be calculated as:

a) Flash temperature criterion
\[ T_{MB} = T_{oil} + 0,5 X_s X_{mp} \theta_{flmean} \]

b) Integral temperature criterion
\[ T_{MC} = T_{oil} + 0,7 X_s X_{mp} \theta_{flint} \]

where:
- \( X_s \) = lubrication factor.
- \( X_{mp} \) = contact factor.
- \( X_{mp} = 0,5 (1 + n_p) \)
- \( n_p \) = number of mesh contact on the pinion (for small gear ratio, the number of wheel meshes should be used if higher).
- \( \theta_{flmean} \) = average of the integrated flash temperature (see 4.4) along the path of contact.
- \( \theta_{flint} = \) mean flash temperature along the path of contact (see 4.5).

4.4 The Flash Temperature \( \theta_{fl} \)

4.4.1 Basic formula
The local flash temperature \( \theta_{fl} \) may be calculated as

\[ \theta_{fl} = 0,325 \mu X_{corr} \omega_{BP} X_{\Gamma}^{3/4} F^{1/2} \frac{P_1 - P_2}{\rho_{cry}} \]

(For bevel gears, replace \( u \) with \( u_v \))

and is to be calculated stepwise along the path of contact from A to E.

where:
- \( \mu \) = coefficient of friction, see 4.3.1.
- \( X_{corr} \) = correction factor taking empirically into account the increased scuffing risk in the beginning of the approach path, due to mesh starting without any previously built up oil film and possible shuffling away oil before meshing if insufficient tip relief.
\[ \Gamma_E = \frac{\sqrt{(d_{c1}/d_{v1})^2 - 1}}{\tan \alpha_{vl}} - 1 \]

At lower point of single pair contact

**Cylindrical gears**

\[ \Gamma_B = \Gamma_E = \frac{2 \pi}{z_1 \tan \alpha_{vl}} \]

**Bevel gears**

\[ \Gamma_B = \Gamma_E = \frac{2 \pi}{z_1 \tan \alpha_{vl}} \]

At upper point of single pair contact

**Cylindrical gears**

\[ \Gamma_D = \Gamma_A + \frac{2 \pi}{z_1 \tan \alpha_{vl}} \]

**Bevel gears**

\[ \Gamma_D = \Gamma_A + \frac{2 \pi}{z_1 \tan \alpha_{vl}} \]

At pitch point \( \Gamma_C = 0 \).

The points F and G (only applicable to cylindrical gears) limiting the extension of tip relief are at

\[ \Gamma_F = \frac{\Gamma_A + \Gamma_B}{2} \]

\[ \Gamma_G = \frac{\Gamma_D + \Gamma_E}{2} \]

**4.4.3 Load sharing factor \( X_\Gamma \)**

The load sharing factor \( X_\Gamma \) accounts for the load sharing between the various pairs of teeth in mesh along the path of contact.

\( X_\Gamma \) is to be calculated stepwise from A to E, using the parameter \( \Gamma_y \).

**4.4.3.1 Gears with \( \beta = 0 \) and no tip relief**

Applicable to both cylindrical and bevel gears.

\[ X_{\Gamma y} = \frac{1}{3} + \frac{1}{3} \frac{\Gamma_E - \Gamma_B}{\Gamma_B - \Gamma_A} \quad \text{for} \quad \Gamma_A \leq \Gamma_y < \Gamma_B \]

\[ X_{\Gamma y} = 1 \quad \text{for} \quad \Gamma_B \leq \Gamma_y \leq \Gamma_D \]

**4.4.3.2 Cylindrical gears with \( \beta = 0 \) and tip relief**

Tip relief on the pinion reduces \( X_\Gamma \) in the range \( G - E \) and increases correspondingly \( X_\Gamma \) in the range \( F - B \).

Tip relief on the wheel reduces \( X_\Gamma \) in the range \( A - F \) and increases correspondingly \( X_\Gamma \) in the range \( D - G \).

Following remains generally:

\[ X_{\Gamma y} = 1 \quad \text{for} \quad \Gamma_B \leq \Gamma_y \leq \Gamma_D \]

\[ X_{\Gamma F} = X_{\Gamma G} = 1/2 \]

In the following it must be distinguished between \( C_a < C_{\text{eff}} \) resp. \( C_a > C_{\text{eff}} \). This is shown by an example below where \( C_{a1} < C_{\text{eff}} \) and \( C_{a2} > C_{\text{eff}} \).

**Range \( A - F \)**

For \( C_{a2} \leq C_{\text{eff}} \)

\[ X_{\Gamma y} = \frac{1}{3} \left( 1 - \frac{C_{a2}}{C_{\text{eff}}} \right) \]

\[ X_{\Gamma y} = X_{\Gamma A} + \frac{\Gamma_y - \Gamma_A}{\Gamma_F - \Gamma_A} \left( \frac{1}{3} + \frac{C_{a2}}{3 C_{\text{eff}}} \right) \quad \text{for} \quad \Gamma_A \leq \Gamma_y \leq \Gamma_F \]

For \( C_{a2} \geq C_{\text{eff}} \)

\[ X_{\Gamma y} = 0 \quad \text{for} \quad \Gamma_A \leq \Gamma_y \leq \Gamma_{A'} \]

with \( \Gamma_{A'} = \Gamma_A + \left( \Gamma_F - \Gamma_A \right) \left( \frac{C_{a2}}{C_{\text{eff}}} - \frac{1}{2} \right) \)

\[ X_{\Gamma y} = 1 - \frac{C_{a2}}{C_{\text{eff}}} + \frac{\Gamma_y - \Gamma_A}{\Gamma_F - \Gamma_A} \left( \frac{C_{a2}}{C_{\text{eff}}} - \frac{1}{2} \right) \quad \text{for} \quad \Gamma_{A'} \leq \Gamma_y \leq \Gamma_F \]

**Range \( F - B \)**

For \( C_{a1} \leq C_{\text{eff}} \)
\[ X_{\gamma'} = \frac{1}{2} + \frac{\Gamma_y - \Gamma_F}{\Gamma_B - \Gamma_F} \left( \frac{1}{6} + \frac{C_{a1}}{3 C_{eff}} \right) \text{ for } \Gamma_F \leq \Gamma_y \leq \Gamma_B \]

For \( C_{a1} \geq C_{eff} \)

\[ X_{\gamma'} = \frac{1}{2} + \frac{\Gamma_y - \Gamma_F}{\Gamma_B - \Gamma_F} \left( \frac{C_{a1}}{C_{eff}} \right) \frac{1}{2} \text{ for } \Gamma_F \leq \Gamma_y \leq \Gamma_B' \]

\[ X_{\gamma'} = 1 \text{ for } \Gamma_B' \leq \Gamma_y \leq \Gamma_B \]

with \( \Gamma_B' = \Gamma_F + \frac{\Gamma_B - \Gamma_F}{2 C_{eff}} \)

Range \( D-G \)

For \( C_{a2} \leq C_{eff} \)

\[ X_{\gamma'} = \frac{2}{3} + \frac{C_{a2}}{3 C_{eff}} \frac{\Gamma_y - \Gamma_D}{\Gamma_G - \Gamma_D} \left( \frac{1}{6} + \frac{C_{a2}}{3 C_{eff}} \right) \text{ for } \Gamma_D \leq \Gamma_y \leq \Gamma_G \]

For \( C_{a2} \geq C_{eff} \)

\[ X_{\gamma'} = 1 \text{ for } \Gamma_D \leq \Gamma_y \leq \Gamma_D' \]

with \( \Gamma_D' = \Gamma_D + (\Gamma_G - \Gamma_D) \frac{C_{a2}}{C_{eff}} - 1 \)

\[ X_{\gamma'} = \frac{C_{a2}}{C_{eff}} - \frac{\Gamma_y - \Gamma_D}{\Gamma_G - \Gamma_D} \left( \frac{C_{a2}}{C_{eff}} \right) \frac{1}{2} \text{ for } \Gamma_D' \leq \Gamma_y \leq \Gamma_G \]

Range \( G-E \)

For \( C_{a1} \leq C_{eff} \)

\[ X_{\gamma'} = \frac{1}{3} \left( 1 - \frac{C_{a1}}{C_{eff}} \right) \]

\[ X_{\gamma'} = \frac{1}{2} - \frac{\Gamma_y - \Gamma_G}{\Gamma_E - \Gamma_G} \left( \frac{1}{6} + \frac{C_{a1}}{3 C_{eff}} \right) \text{ for } \Gamma_G \leq \Gamma_y \leq \Gamma_E \]

For \( C_{a1} \geq C_{eff} \)

\[ X_{\gamma'} = \frac{1}{2} - \frac{\Gamma_y - \Gamma_G}{\Gamma_E - \Gamma_G} \left( \frac{C_{a1}}{C_{eff}} \right) \frac{1}{2} \text{ for } \Gamma_G \leq \Gamma_y \leq \Gamma_E' \]

\[ X_{\gamma'} = 0 \text{ for } \Gamma_E' \leq \Gamma_y \leq \Gamma_E \]

with \( \Gamma_E' = \Gamma_E + \frac{\Gamma_E - \Gamma_G}{2 C_{a1}} \frac{1}{C_{eff}} \)

4.4.3.3 Gears with \( \beta > 0 \), buttressing

Due to oblique contact lines over the flanks, a certain buttressing may occur near \( A \) and \( E \).

This applies to both cylindrical and bevel gears with tip relief \( < C_{eff} \). The buttressing \( X_{but} \) is simplified as a linear function within the ranges \( A-H \) resp. \( I-E \).

\[ X_{but}, \text{ for } \Gamma_y < \Gamma_H \text{ and } \Gamma_y > \Gamma_I \]

\[ X_{but} = 1 + 0.3 \epsilon \beta \text{ when } \epsilon \beta < 1 \]

\[ X_{but} = 1.3 \text{ when } \epsilon \beta \geq 1 \]

Cylindrical gears

\[ \Gamma_H - \Gamma_A = \Gamma_E - \Gamma_I = 0.2 \sin \beta_h \]

Bevel gears

\[ \Gamma_H - \Gamma_A = \Gamma_E - \Gamma_I = 0.2 \sin \beta_{bm} \]

4.4.2.4 Gears with \( \epsilon \gamma \leq 2 \) and no tip relief

Applicable to both cylindrical and bevel gears.

\[ X_{\gamma'} \text{ is obtained by multiplication of } X_{\gamma'} \text{ in 4.4.3.1 with } X_{but} \text{ in 4.4.3.3.} \]

4.4.3.5 Gears with \( \epsilon \gamma > 2 \) and no tip relief

Applicable to both cylindrical and bevel gears.

\[ X_{\gamma'} = \frac{1}{\epsilon_a} X_{but} \text{ for } \Gamma_y < \Gamma_H \text{ and } \Gamma_y > \Gamma_I \]

\[ X_{\gamma'} = \frac{1}{\epsilon_a} X_{but} \text{ for } \Gamma_y < \Gamma_H \text{ and } \Gamma_y > \Gamma_I \]

4.4.3.6 Cylindrical gears with \( \epsilon \gamma \leq 2 \) and tip relief

\[ X_{\gamma'} \text{ is obtained by multiplication of } X_{\gamma'} \text{ in 4.4.3.2 with } X_{but} \text{ in 4.4.3.3.} \]

4.4.3.7 Cylindrical gears with \( \epsilon \gamma > 2 \) and tip relief

Tip relief on the pinion (resp. wheel) reduces \( X_{\gamma'} \) in the range \( G-E \) (resp. \( A-F \)) and increases \( X_{\gamma'} \) in the range \( H-G \).
$X_r$ is obtained by multiplication of $X_r$ as described below with $X_{br}$ in 4.4.3.3.

In the $X_r$ example below the influence of tip relief is shown (without the influence of $X_{br}$) by means of $C_{a1} > C_{eff}$ and $C_{a2} < C_{eff}$.

Tip relief $> C_{eff}$ causes new end points $A'$ resp. $E'$ of the path of contact.

For $C_{a1} = C_{a2} = C_{eff}$ the following applies:

\[
X_{\Gamma_y} = \frac{C_{eff} - C_{a2}}{\epsilon_a C_{eff}} + \frac{C_{a2} - C_{eff}}{\epsilon_a (\epsilon_a - 1) C_{a1} + (3 \epsilon_a + 1) C_{a2}}
\]

\[
\frac{\Gamma_y - \Gamma_A}{\Gamma_E - \Gamma_A} \frac{(\epsilon_a - 1) C_{a1} + (3 \epsilon_a + 1) C_{a2}}{2 \epsilon_a (\epsilon_a + 1) C_{eff}}
\]

for $\Gamma_A \leq \Gamma_y \leq \Gamma_E$ if $C_{a2} \leq C_{eff}$

and for $\Gamma_A \leq \Gamma_y \leq \Gamma_E$ if $C_{a2} > C_{eff}$

\[
X_{\Gamma_y} = 0 \quad \text{for} \quad \Gamma_A \leq \Gamma_y \leq \Gamma_A' \quad \text{if} \quad C_{a2} > C_{eff}
\]

with $\Gamma_A' = \Gamma_A + (\Gamma_E - \Gamma_A) \frac{(C_{a2} - C_{eff}) (2 \epsilon_a + 1)}{(\epsilon_a - 1) C_{a1} + (3 \epsilon_a + 1) C_{a2}}$

\[X_{\Gamma_y} = \frac{1}{\epsilon_a} + \frac{(\epsilon_a - 1) (C_{a1} + C_{a2})}{2 \epsilon_a (\epsilon_a + 1) C_{eff}} \quad \text{for} \quad \Gamma_F \leq \Gamma_y \leq \Gamma_G
\]

\[X_{\Gamma_y} = X_{\Gamma_y-\Gamma_G} - \frac{\Gamma_y - \Gamma_G}{\Gamma_E - \Gamma_G} \frac{(3 \epsilon_a + 1) C_{a1} + (\epsilon_a - 1) C_{a2}}{2 \epsilon_a (\epsilon_a + 1) C_{eff}} \]

for $\Gamma_G \leq \Gamma_y \leq \Gamma_E$ if $C_{a1} \leq C_{eff}$

and for $\Gamma_G \leq \Gamma_y \leq \Gamma_E$ if $C_{a1} > C_{eff}$

\[
X_{\Gamma_y} = 0 \quad \text{for} \quad \Gamma_E' \leq \Gamma_y \leq \Gamma_G \quad \text{if} \quad C_{a1} \geq C_{eff}
\]

with $\Gamma_E' = \Gamma_E - (\Gamma_E - \Gamma_G) \frac{(C_{a1} - C_{eff}) (2 \epsilon_a + 1)}{(3 \epsilon_a + 1) C_{a1} + (\epsilon_a - 1) C_{a2}}$

4.4.3.8 Bevel gears with $\epsilon_y > 2$ and tip relief

For $C_{a1} = C_{a2} = C_{eff}$ the following applies:

\[
X_{\Gamma_y} = 0.45 X_{\Gamma_y}(C_{a2} = 0) + 0.55 X_{\Gamma_y}(C_{a2} = C_{eff})
\]

\[
X_{\Gamma_y} = 0.6 X_{\Gamma_y}(C_{a1} = 0) + 0.4 X_{\Gamma_y}(C_{a1} = C_{eff})
\]

\[
X_{\Gamma_y} = 1.5 X_{\Gamma_y}(C_{a1} = C_{eff}) - \frac{3}{4 - \frac{C_{a1}}{C_{eff}}} \left(1 - \frac{\Gamma_y - \Gamma_M}{\Gamma_A' - \Gamma_M} \right)
\]

\[
X_{\Gamma_y} = 1.5 X_{\Gamma_y}(C_{a1} = C_{eff}) - \frac{3}{4 - \frac{C_{a1}}{C_{eff}}} \left(1 - \frac{\Gamma_y - \Gamma_M}{\Gamma_E' - \Gamma_M} \right)
\]
Range E'—E

\[ X_{\Gamma y} = 0 \]

4.4.3.9 Bevel gears with \( \varepsilon_\gamma \leq 2 \) and tip relief

For tip relief \( \leq C_{eff} \), \( X_{\Gamma y} \) is found by linear interpolation between \( X_{\Gamma y} (C_a = 0) \) as in 4.4.3.4 and \( X_{\Gamma y} (C_a = C_{eff}) \) as in 4.4.3.8.

The interpolation is also described in 4.4.3.8.

For tip relief \( \geq C_{eff} \), \( X_{\Gamma y} \) is as in 4.4.3.8. Note that the ranges A—M and M—E have to be treated separately.

4.5 The mean Flash Temperature \( \theta_{\text{frain}} \)

(Integral criterion)

**Cylindrical gears**

\[ \theta_{\text{frain}} = 50 \mu X_{BE} \frac{\omega_{BE}^{3/2} v_1^{1/2}}{|a_1|^{1/4}} X_\gamma \frac{X_e}{X_{Ca}} \]

**Bevel gears**

\[ \theta_{\text{frain}} = 50 \mu X_{BE} \frac{\omega_{BE}^{3/2} v_1^{1/2}}{|a_v|^{1/4}} X_\gamma \frac{X_e}{X_{Ca}} \]

\( \mu = \) see 4.3.1.

\( X_{BE} = \) geometry factor at point E.

\( X_{Ca} = \) contact ratio factor.

Cylindrical gears

\[ X_{BE} = 0.5 \sqrt{|u_1 + 1|} \left( \frac{\rho_{E1} - \sqrt{\frac{\rho_{E2}}{u}}}{\rho_{E1} \rho_{E2}} \right)^{1/4} \]

\[ \rho_{E1} = 0.5 \sqrt{d_{a1}^2 - d_{b1}^2} \]

\[ \rho_{E2} = a \sin \alpha_{tw} - \rho_{E1} \]

Bevel gears

\[ X_{BE} = 0.5 \sqrt{u_v + 1} \left( \frac{\rho_{E1} - \sqrt{\frac{\rho_{E2}}{u_v}}}{\rho_{E1} \rho_{E2}} \right)^{1/4} \]

\[ \rho_{E1} = 0.5 \sqrt{d_{a1}^2 - d_{b1}^2} \]

\[ \rho_{E2} = a_v \sin \alpha_{tv} - \rho_{E1} \]

\( \omega_{BE} = \) see 4.3.1 but additionally the factor \( K_{g\gamma} \) applies.

\[ K_{\delta\gamma} = 1 \quad \text{for} \quad \varepsilon_\gamma \leq 2 \]

\[ K_{\delta\gamma} = 1 + 0.2 \sqrt{\left(\varepsilon_\gamma - 2\right) (5 - \varepsilon_\gamma)} \quad \text{for} \quad 2 < \varepsilon_\gamma \leq 3.5 \]

\[ K_{\delta\gamma} = 1.3 \quad \text{for} \quad \varepsilon_\gamma \geq 3.5 \]

\( X_e = \) tip relief factor.

**Cylindrical gears**

\[ \varepsilon_1 = \frac{z_1}{2\pi} \left( \sqrt{\left( \frac{d_{a1}}{d_{b1}} \right)^2 - 1} - \tan \alpha_{tw} \right) \]

\[ \varepsilon_2 = \varepsilon_a - \varepsilon_1 \]

For \( 1 \leq \varepsilon_a < 2, \varepsilon_1 < 1, \varepsilon_2 < 1 \)

\[ X_e = \frac{0.7 (-\varepsilon_2^2 + \varepsilon_2)}{e_1 e_2} - 0.22 \varepsilon_a - 0.52 - 0.6 \varepsilon_1 \varepsilon_2 \]

For \( 1 \leq \varepsilon_a < 2, \varepsilon_1 \geq 1, \varepsilon_2 < 1 \)

\[ X_e = \frac{0.18 \varepsilon_2^2 + 0.7 \varepsilon_2^2 + 0.82 \varepsilon_1 - 0.52 \varepsilon_2 - 0.3 \varepsilon_1 \varepsilon_2}{\varepsilon_a \varepsilon_1} \]

For \( 1 \leq \varepsilon_a < 2, \varepsilon_1 < 1, \varepsilon_2 \geq 1 \)

\[ X_e = \frac{0.7 \varepsilon_2^2 + 0.18 \varepsilon_2 - 0.52 \varepsilon_1 + 0.82 \varepsilon_2 - 0.3 \varepsilon_1 \varepsilon_2}{2 \varepsilon_a \varepsilon_1} \]

For \( 2 \leq \varepsilon_a < 3, \varepsilon_1 \geq \varepsilon_2 \)

\[ X_e = \frac{0.44 \varepsilon_2^2 + 0.59 \varepsilon_2 + 0.3 \varepsilon_1 - 0.3 \varepsilon_2 - 0.15 \varepsilon_1 \varepsilon_2}{2 \varepsilon_a \varepsilon_1} \]

For \( 2 \leq \varepsilon_a < 3, \varepsilon_1 < \varepsilon_2 \)

\[ X_e = \frac{0.5 \varepsilon_2^2 + 0.44 \varepsilon_2 - 0.3 \varepsilon_1 + 0.3 \varepsilon_2 - 0.15 \varepsilon_1 \varepsilon_2}{2 \varepsilon_a \varepsilon_1} \]

**Bevel gears**

As above, but replace \( \varepsilon_1 \) with \( \varepsilon_v \)

\( \varepsilon_2 \) with \( \varepsilon_{v2} \)

\[ X_\gamma = \text{approach factor.} \]

**Cylindrical gears**

\[ X_Q = 1 \quad \text{for} \quad \frac{e_2}{\varepsilon_a} \leq 1.5 \]

\[ X_Q = 1.4 - \frac{4 e_2}{15 \varepsilon_a} \quad \text{for} \quad 1.5 < \frac{e_2}{\varepsilon_a} < 3 \]

\[ X_Q = 0.6 \quad \text{for} \quad 3 \leq \frac{e_2}{\varepsilon_a} \]

with

\( e_1 = e_2 \) and \( \varepsilon_a = \varepsilon_1 \) if the pinion is driving.

\( e_1 = \varepsilon_1 \) and \( \varepsilon_a = \varepsilon_2 \) if the pinion is driven.

**Bevel gears**

As above, but replace

\( \varepsilon_1 \) with \( \varepsilon_v \)

\( \varepsilon_2 \) with \( \varepsilon_{v2} \)

**Cylindrical gears**

\[ X_{Ca} = \text{tip relief factor.} \]

For accuracy grade \( \geq 7 \), \( X_{Ca} = 1 \)

For accuracy grade \( \leq 6 \):

\[ X_{Ca} = 1 + \left( 0.06 + 0.18 \frac{C_a}{C_{eff}} \right) e_{\text{max}} + \left( 0.02 + 0.69 \frac{C_a}{C_{eff}} \right) e_{\text{max}}^2 \]

\( C_{eff} = \) see 4.3.2.

\( C_a = \) nominal tip relief for a.m. formula.
If the pinion is driving and $\epsilon_1 > 1.5 \epsilon_2$
\[
C_a = C_{a1} \quad \text{for} \quad C_{a1} \leq C_{\text{eff}}
\]
\[
C_a = C_{\text{eff}} \quad \text{for} \quad C_{a1} > C_{\text{eff}}
\]
If the pinion is driven and $\epsilon_1 \leq 1.5 \epsilon_2$
\[
C_a = C_{a2} \quad \text{for} \quad C_{a2} \leq C_{\text{eff}}
\]
\[
C_a = C_{\text{eff}} \quad \text{for} \quad C_{a2} > C_{\text{eff}}
\]
If the pinion is driven and $\epsilon_1 \geq (2/3) \epsilon_2$
\[
C_a = C_{a1} \quad \text{for} \quad C_{a1} \leq C_{\text{eff}}
\]
\[
C_a = C_{\text{eff}} \quad \text{for} \quad C_{a1} > C_{\text{eff}}
\]

If the pinion is driven and $\epsilon_1 < (2/3) \epsilon_2$
\[
C_a = C_{a2} \quad \text{for} \quad C_{a2} \leq C_{\text{eff}}
\]
\[
C_a = C_{\text{eff}} \quad \text{for} \quad C_{a2} > C_{\text{eff}}
\]

Bevel gears
As above, but replace $\epsilon_{\text{max}}$ with $\epsilon_{v_{\text{max}}}$ where $\epsilon_{v_{\text{max}}}$ is max of $\epsilon_{v1}$ and $\epsilon_{v2}$.
Appendix A  Fatigue Damage Accumulation

The Palmgren-Miner cumulative damage calculation principle is used. The procedure may be applied as follows:

A.1 Stress Spectrum

From the individual torque classes, the torques (T_i) at the peak values of class intervals and the associated number of cycles (N_{ri}) for both pinion and wheel are to be listed from the highest to the lowest torque. (In case of a cyclic torque variation within the torque classes, it is advised to use the peak torque. If the cyclic variation is such that the same teeth will repeatedly suffer the peak torque, this is a must.)

The stress spectra for tooth roots and flanks (σ_{fr}, σ_{fl}) with all relevant factors (except K_A) are to be calculated on the basis of the torque spectrum. The load dependent K-factors are to be determined for each torque class.

A.2 a—N-curve

The stress versus load cycle curves for tooth roots and flanks (both pinion and wheel) are to be drawn on the basis of permissible stresses (i.e. including the demanded minimum safety factors) as determined in 2 resp. 3. If different safety levels for high cycle fatigue and low cycle fatigue are desired, this may be expressed by different demand safety factors applied at the endurance limit resp. at static strength.

A.3 Damage accumulation

The individual damage ratio D_i at ith stress level is defined as

\[ D_i = \frac{N_{ri}}{N_{fi}} \]

where:

- N_{ri} = the number of applied cycles at ith stress.
- N_{fi} = the number of cycles to failure at ith stress.

Basically stresses σ_i below the permissible stress level for infinite life (if a constant Z_N or Y_N is accepted) do not contribute to the damage sum. However, calculating the actual safety factor S_{act} as described below all the σ_i for which the product S σ_i is bigger than or equal to the permissible stress level for infinite life contribute to the damage sum and thus to the determination of S_{act}. The final value of S is decisive.

(N_{fi} can be found mathematically by putting the permissible stress σ_e equal the actual stress σ_i, thereby finding the actual life factor. This life factor can be solved with regard to load cycles, i.e. N_{fi}.)

The damage sum \( \Sigma D_i \) is not to exceed unity.

If \( \Sigma D_i \neq 1 \), the safety against cumulative fatigue damage is different from the applied demand safety factor. For determination of this theoretical safety factor an iteration procedure is required as described in the following flowchart:

- Multiply all stresses \( \sigma \) with a factor S
  - Reduce S
    - Calculate the new minsum \( \Sigma D_i \)
      - Yes \( \Sigma D_i > 1,05 \)
      - No \( \Sigma D_i < 0,95 \)
        - Yes
          - Output S
        - No
          - Increase S

S is a correction factor with which the actual safety factor S_{act} can be found.

S_{act} is the demand safety factor (used in determination of the permissible stresses in the \( \sigma - N \) - curve) times the correction factor S.

The full procedure is to be applied for pinion and wheel, tooth roots and flanks.

Note:

If alternating stresses occur in a spectrum of mainly pulsating stresses, the alternating stresses may be replaced by equivalent pulsating stresses, i.e. by means of division with the actual design factor Y_d.